Analysis of Pile Groups Subjected to Vertical and Horizontal Loads

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Summary—Three methods of analysing the behaviour of pile groups are described, a well established statical method in which no account is taken of pile-soil interaction, a method in which the pile group is replaced by an equivalent structural bent and a method based on elastic theory in which interaction between the piles is taken into account in a logical manner. Comparisons between these three methods indicate that consideration of inter-pile interaction in the soil leads to increased maximum loads and moments in a group, although the deflections and rotations may not differ greatly.

A parametric study is made of the deflections and rotation of typical pile groups, using the elastic interaction method. The effects of pile batter, increased pile spacing and increased pile stiffness in decreasing the group deflections and rotation are examined.

SYMBOLS

A	cross-sectional area of pile
A_e	equivalent cross-sectional area of pile
$\mathbf{A}_v, \mathbf{A}_h, \mathbf{A}_\theta$; $\mathbf{B}_v, \mathbf{B}_h, \mathbf{B}_\theta$; $\mathbf{C}_v, \mathbf{C}_h, \mathbf{C}_\theta$	submatrices in elastic interaction method
E_p	Young's modulus of pile
E_s	Young's modulus of soil
H	horizontal load on pile group
H_i	horizontal load on pile i
I	settlement influence factor for single pile
I_p	moment of inertia of pile
$I_{\rho H}, I'_{\rho H}$	horizontal displacement influence factors due to horizontal load
$I_{\rho M}, I'_{\rho M}$	horizontal displacement influence factors for moment
$I_{\rho F}, I'_{\rho F}$	horizontal displacement influence factors for fixed head pile
$I_{\theta H}, I'_{\theta H}$	rotation influence factors for horizontal load
$I_{\theta M}, I'_{\theta M}$	rotation influence factors for moment (note: primed values are for linearly increasing E_s with depth)
I_{vv}, I_{vH}, I_{vM}, etc	displacement and rotation factors for a group of piles
K	pile stiffness factor
K_R	pile flexibility factor (constant E_s)
K_N	pile flexibility factor (linearly varying E_s)
L	embedded length of pile
L_b	equivalent total length of pile
L_e	equivalent length of embedded portion of pile
M	moment acting on pile group
M_i	moment at head of pile i
N_h	rate of increase of Young's modulus with depth
R_A	area ratio of pile
R_s	group settlement ratio
$R_{\rho H}, R_{\rho M}, R_{\rho F}$	group displacement ratios for horizontal load, moment and for fixed head piles
$R_{\theta H}, R_{\theta M}$	group rotation ratios for horizontal load and moment
V	vertical load acting on group
V_i	vertical load on pile i
d	pile diameter
e	unsupported length of pile above ground surface
n	number of piles in group
s	centre to centre spacing of piles
s_e	equivalent value of s for battered piles
x_i	distance from centre of gravity of pile group to pile i, in the positive x direction
α	interaction factor for vertical loading
$\alpha_{\rho H}, \alpha_{\rho M}, \alpha_{\rho F}$	interaction factors for horizontal displacement due to horizontal load, moment and for fixed head pile
$\alpha_{\theta H}, \alpha_{\theta M}$	interaction factors for rotation due to horizontal load and moment
β	departure angle between two piles
ν_s	Poisson's ratio of soil
ρ	single pile settlement
ρ_v	settlement of pile group
ρ_h	horizontal movement of pile group
$\rho_{a1}, \rho_{NQ1}, \rho_{NM1}$	single pile vertical and horizontal movements due to unit loads and moment
θ	rotation
θ_{N1}, θ_{M1}	single pile rotations due to unit load and moment

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INTRODUCTION

A considerable number of methods have been developed for analysing the behaviour of pile groups subjected to a general loading system. Such methods may be classified into three categories:

(i) simple statical methods which ignore the presence of the soil and consider the pile group as a purely structural system.

(ii) methods which reduce the pile group to a structural system but which take some account of the effect of the soil by determining equivalent free-standing lengths of the piles. The theory of subgrade reaction is generally used to determine these equivalent lengths. Typical of these methods are those described in Francis (1964), Hrennikoff (1950), Kocsis (1968), Nair et al (1969) and Priddle (1963). This type of approach will be termed the "equivalent bent method", following Kocsis (1968).

(iii) a method in which the soil is assumed to be an elastic continuum and inter-pile interaction can be considered.

The first two methods can only consider interaction of the piles through the pile cap and not inter-pile interaction through the soil as well. They therefore assume that, once the loads on any pile are known, the deflections of that pile may be calculated from these loads alone. The third method removes this limitation and allows consideration of pile interaction through the soil; the deflections of a pile are therefore not only a function of the loads on that pile but also of the loads on all the piles in the group.

In this paper the above three approaches will be described and comparisons will be presented between the results of these analyses for two typical examples. In the "equivalent bent method", a modified method of determining equivalent free-standing lengths, based on elastic theory rather than subgrade reaction theory, is detailed. Finally some parametric solutions obtained from the third method will be presented for typical pile groups, to illustrate the effects of various factors on group behaviour.

SIMPLE STATICAL ANALYSIS

Traditional design methods have relied on the consideration of the pile group as a purely structural system, ignoring the effect of the soil. One such method which may be employed either graphically or analytically, is illustrated in Fig. 1. Considering, for simplicity, loads and batter in the x, z plane only and piles having a pinned head, the steps in this method are as follows

(*a*) the vertical pile loads are calcul ated as

$$V_i = \frac{V}{n} + \frac{M x_i}{\sum_{j=1}^{n} x_j^2} \qquad (1)$$

where x_i, x_j are distance to the heads of piles i and j, measured from the centre of gravity of the group (i.e. the point about which $\sum_{i=1}^{n} x_i = 0$).

(*b*) if the solution is done graphically, the forces V and H are plotted on a force polygon. The vertical pile forces V_i from (1) are then set off.

(*c*) the force polygon is then completed by drawing lines parallel to the pile directions. The axial force in each pile may thus be obtained. There is then a residual horizontal force H_e which is assumed to be equally distributed between each pile in the group.

(*d*) if desired the design of the group may be amended and the pile batters adjusted to give $H_e = 0$ i.e. no normal component of load in the piles, so that each pile is axially loaded.

It should be noted that this method cannot take account of different conditions of fixity at the pile head and always assumes zero moment at the head of each pile.

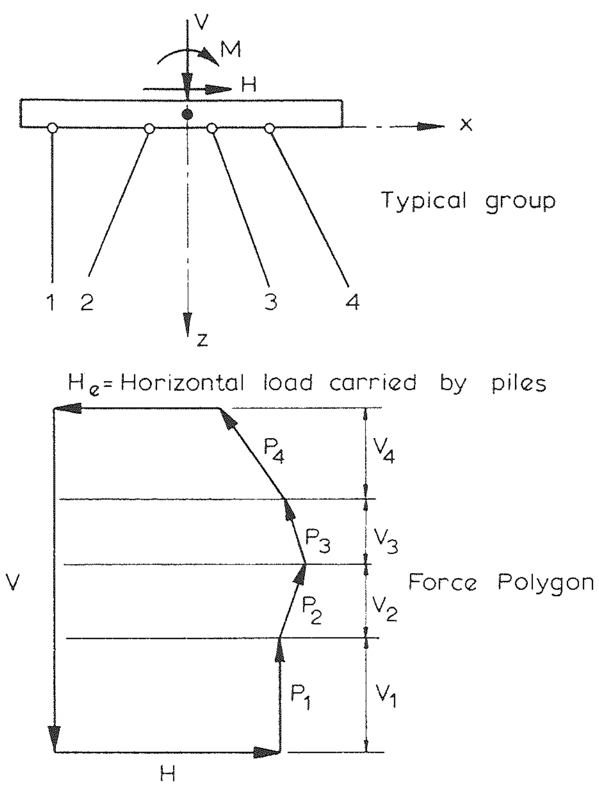


Fig. 1—Approximate method for determination of group load distribution.

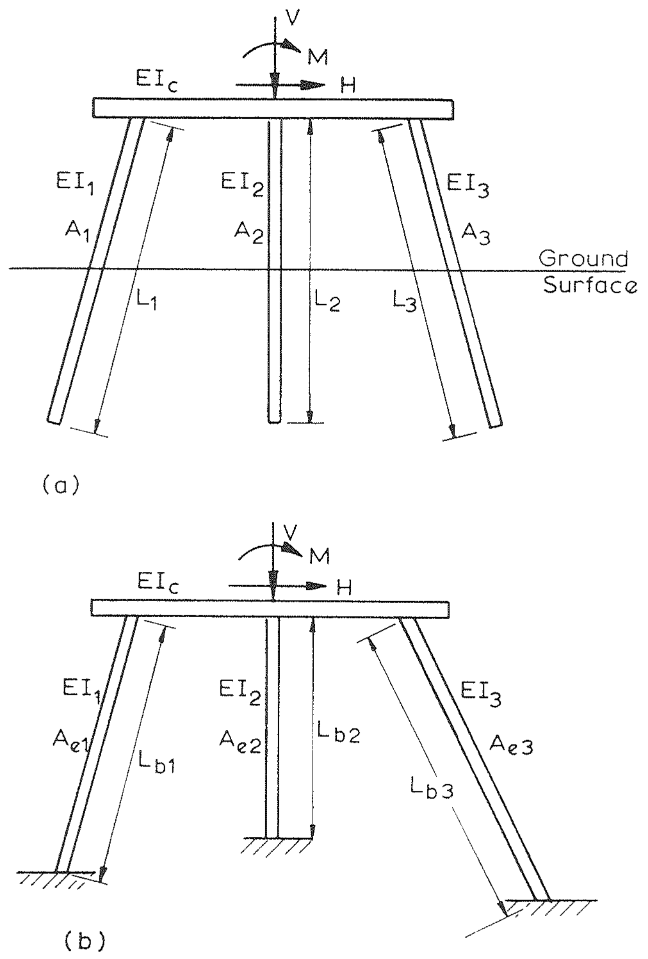


Fig. 2—Principle of equivalent bent approach.
(a) Actual pile group.
(b) Equivalent Bent.

EQUIVALENT BENT METHOD

Principle of method

The principle of this method is illustrated in Fig. 2 for a planar group. The actual group is shown in Fig. 2a and is acted up by vertical and horizontal forces and a moment. The equivalent bent is shown in Fig. 2b and consists of the pile cap supported by fixed-ended free-standing columns of total equivalent lengths L_1, L_2 and L_3 and equivalent cross-sectional areas A_{e1}, A_{e2} and A_{e3}. The equivalent area of a pile is such that the axial deflection of the actual pile is equal to the axial deflection of the equivalent column while the equivalent length is such that equal lateral deflections or rotations are obtained. Once the equivalent lengths and areas have been determined, the equivalent bent may be analysed by standard structural analysis techniques to determine the deflections, rotations and pile loads in the system.

Determination of equivalent bent

In existing methods which use the above approach, the equivalent lengths of the piles are almost invariably determined from a subgrade reaction analysis. The normal deflection (or rotation) of a pile subjected to normal load or moment is calculated and equated to the normal deflection (or rotation) of a cantilever under the same load or moment, from which the equivalent length can be determined (e.g. Francis (1964), Kocsis (1968) and Nair et al (1969)). The equivalent area of each pile has generally been determined by equating the axial deformation of the equivalent cantilever to the free-standing column axial deformation of the actual pile i.e. interaction with the soil has been ignored.

With the development of elastic solutions for vertically- and laterally-loaded piles (Poulos (1971a); Poulos (1971b); Poulos & Mattes (1971); Poulos (1972); and Poulos (unpublished)) it is now possible to make a more satisfactory determination of the equivalent lengths and areas of the piles in the equivalent bent method, taking rational account of the effects of group action. This method of determination is described below.

(a) Equivalent length of piles

The simplest basis for determining the equivalent cantilever length of the embedded portion of a pile is to equate the lateral deflection of the pile at the ground line and of the equivalent cantilever at the ground line, (alternatively, the corresponding rotations could be equated, but Nair et al (1969) and Kocsis (1968) found only small differences in the equivalent length determined by the two approaches).

The equivalent length of the pile will depend on the boundary condition at the pile head and on the type of loading assumed to act. A number of cases have been considered, as illustrated in Fig. 3, and the solutions derived for the equivalent cantilever lengths are summarised in Table I.

TABLE I

Expressions for equivalent cantilever lengths*

Case (see Fig. 3)	Equivalent Length
a	$L_{eH} = L\sqrt[3]{3I_{\rho H}K_R R_{\rho H}}$
b	$L_{eM} = L\sqrt{2I_{\rho M}K_R R_{\rho M}}$
c	$L_{eF} = L\sqrt[3]{12K_R I_{\rho F}R_{\rho F}}$
d & e	L_e is solution to equation $\left(\frac{L_e}{L}\right)^3 + 1.5\frac{M}{HL}\left(\frac{L_e}{L}\right)^2 = 3K_R(R_{\rho H}I_{\rho H} + \frac{M}{HL}I_{\rho M}R_{\rho M})$ For case d, $(L_e = L_{e1})$ For case e, $(L_e = L_{e2})$ $M = He$ $M = -HL\left[\frac{I_{\theta H}.K_R.R_{\theta H} + 1/6(e/L)^2}{I_{\theta M}.K_R.R_{\theta M} - 1}\right] + He$

*Above solutions are for constant E_s. For linearly increasing E_s, replace K_R by K_N, and influence factors $I_{\rho H}$ etc. by $I'_{\rho H}$ etc.

$I_{\rho H}$, $I_{\rho M}$, $I_{\rho F}$, $I_{\theta H}$ and $I_{\theta M}$ are displacement and rotation influence factors (see symbols), and K_R is the pile flexibility factor, defined as

$$K_R = \frac{E_p I_p}{E_s L^4} \qquad (2)$$

where E_pI_p pile stiffness

E_s Young's modulus of the soil (assumed constant with depth)

L embedded length of pile.

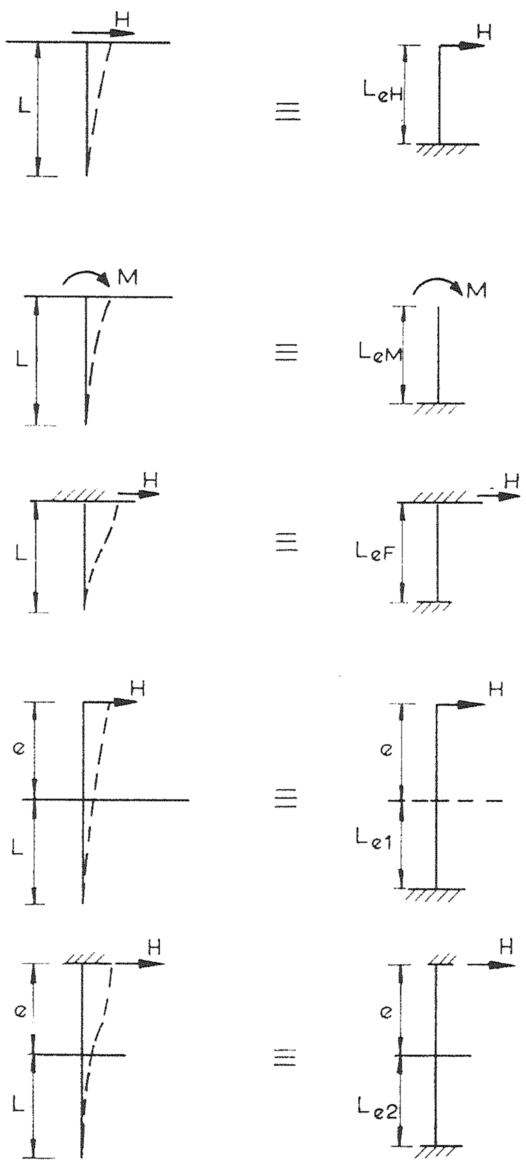
Values of the influence factors have been obtained by Poulos (1971a). The group effect has been taken into account approximately by applying group displacement and rotation ratios $R_{\rho H}$, $R_{\rho M}$, $R_{\rho F}$, $R_{\theta H}$, $R_{\theta M}$ to the single pile movement (Poulos, unpublished).

The above displacement and rotation ratios are determined by superposition of appropriate "interaction factors" for laterally-loaded piles, expressing the increase in deflection or rotation of a pile due to an adjacent loaded pile. Values of these interaction factors and typical values of the group displacement and rotation ratios, are given by Poulos (1971b). Table I gives directly the equivalent lengths for constant Young's modulus E_s with depth; corresponding solutions for linearly increasing Young's modulus with depth may be obtained by replacing the influence factors $I_{\rho H}$ etc. by the appropriate values for a linearly increasing Young's modulus, denoted as $I'_{\rho H}$ etc., and the pile stiffness factor K_R by K_N, where

$$K_N = \frac{E_pI_p}{N_hL^5} \qquad (3)$$

where N_h rate of increase of Young's modulus with depth.

Values of $I'_{\rho H}$ etc. are given in Poulos (1973). Although the interaction factors in Poulos (1971b) apply strictly only to the case of constant



Actual pile *Equivalent cantilever*

Fig. 3—Equivalent cantilevers for laterally loaded piles.

Young's modulus with depth they may also be applied approximately to the case of a linearly increasing modulus with depth, using a value of K_R equal to K_N.

It should be noted that for case (e), the first term of the expression for M represents the fixing moment developed at the pile head. If fixity is not considered to be fully effective, a reduction factor, ranging between 1 and 0, can be applied to this first term. In the limit, if no fixity is developed, case (e) then reduces to case (d).

Table II gives an example of the difference between the equivalent lengths L_{eH} and L_{eM}, assuming lateral load only, and moment only, to act respectively. A single free-head pile only is considered so that $R_{\rho H} = R_{\rho M} = 1$. For flexible piles the equivalent length L_{eH} is greater, but for rigid piles ($K_R > 10^{-1}$), L_{eM} becomes slightly greater.

The derivation of the equivalent bent as described above assumes linear elastic soil response, but as pointed out by Poulos (1973), this may not be a good assumption for lateral loading. An iterative approach can be adopted if desired in which a non-linear load-deflection curve is specified for each pile and the solution from the analysis of the equivalent bent is recycled, using successively corrected values of the equivalent cantilever length, until the load and deflection of each pile are compatible.*

TABLE II

Equivalent lengths L_{eH} and L_{eM}†

K_R	10^{-6}	10^{-5}	10^{-4}	10^{-3}	10^{-2}	10^{-1}
L_{eH}/L	.0406	.0818	.157	.293	.551	1.123
L_{eM}/L	.0251	.0614	.127	.244	.466	1.154

† $L/d = 50$, $\nu_s = 0.5$, single pile, constant E_s.

(b) Equivalent area of piles

For a fully embedded pile, the settlement of a pile in the group is given approximately as

$$\rho = \frac{V}{E_sd} IR_s \qquad (4)$$

where I settlement influence factor

V applied axial load

R_s group settlement ratio.

Solutions for I have been presented by Poulos (1972) and Poulos (1974) while values of R_s have been given by Poulos and Mattes (1971) for a wide range of groups.** Both I and the interaction factors are functions of the pile stiffness factor K, where

$$K = \frac{E_pR_A}{E_s} \qquad (5)$$

where E_p Young's modulus of pile

E_s Young's modulus of soil

R_A area ratio of pile which equals ratio of area of pile section to gross cross-sectional area.

Although the solutions given are for constant E_s with depth, they may be applied approximately to other cases, provided that an average value of E_s along the length of the pile is used.

For estimating R_s when the group contains battered piles, the battered piles can, as a first approximation, be considered as vertical piles located at the mid-point of the embedded part of the pile, and allowance can be made for tensile loads in the piles (which, in effect, will decrease settlement interaction between the piles).

The equivalent pile will have a length L_e which will be determined as described above (Table I) for lateral deflection equivalence. The axial deflection of this equivalent pile is

$$\rho = \frac{VL_e}{E_pA_e} \qquad (6)$$

*In the iterative procedure, it is also possible to successively correct the values of the group displacement and rotation ratios, to allow for any effects of non-uniform load distribution.

**These values apply for groups which are loaded symmetrically and in which all pile loads are compressive, and may not be accurate for groups in which some of the piles are in tension.

where E_p Young's modulus of pile
A_e equivalent area of pile
From (4) and (5),

$$A_e = \frac{L_e d}{IR_s} \frac{E_s}{E_p} \quad (7)$$

For a pile having an unsupported length e above the ground surface, the axial deflection of this length must be added to that for the embedded portion. The corresponding expression for A_e is then

$$A_e = \frac{L_e + e}{\left(\frac{IR_s}{d} \frac{E_p}{E_s} + \frac{e}{A}\right)} \quad (8)$$

The above expressions for A_e should apply for battered piles as well as vertical piles since, as shown by Poulos and Madhav (1971), the axial movement of a pile is not significantly influenced by its inclination.

ELASTIC INTERACTION ANALYSIS

Analyses of the behaviour of groups of vertical piles, based on elastic theory, have been presented for axial loading (Poulos and Mattes, 1971) and lateral loading (Poulos, 1971b).

These analyses have been based on the use of "interaction factors", which express the increase in movement of a pile due to an adjacent loaded pile and which are functions of the pile spacing, relative stiffness and geometry, and for horizontal loads, of the direction of loading. By summation of the interaction factors for each pile in a group due to all the other piles in the group, the displacement of each pile may be written in terms of the loads on each pile in the group.

This approach can be extended to groups containing battered piles. An earlier such extension was described by Poulos and Madhav (1971). The present method contains some revisions of this earlier method. The case considered first is a group in which all the piles are battered in the same plane and on which the horizontal load acts in the same plane. Considering two piles i and j in a group, it is assumed that an axial load on pile j will cause a deflection of pile i which is in the axial direction of pile j and equal to the axial deflection of pile j under this axial load multiplied by an interaction factor for axial loading. Similarly it is assumed that a normal load on pile j will cause a deflection of pile i which is in the normal direction of pile j and equal to the normal deflection of pile j under this normal load multiplied by an interaction factor for normal loading. It will be assumed for simplicity that the interaction factors for two battered piles are identical with those for vertical piles at some equivalent spacing s_e. Calculations suggest that for practical ranges of pile flexibility, s_e is approximately the centre-to-centre distance between the piles one-third of the vertical depth of the pile for lateral loading, and somewhat greater for axial loading. However, for convenience, the same equivalent spacing will be assumed for both axial and lateral loading (see Fig. 4). It is further assumed, following the findings of Poulos and Madhav (1971), that the interaction factor for axial displacement due to axial load equals that for vertical displacement due to vertical load on a vertical pile, and the rotation and normal displacement interaction factors due to normal load and moment are identical with those for rotation and horizontal displacement due to horizontal load and moment.

On the basis of the above assumptions, the resulting equations for vertical and horizontal displacement and rotation may be written in matrix form as follows:

$$\begin{pmatrix} \mathbf{A}_v & \mathbf{B}_v & \mathbf{C}_v \\ \mathbf{A}_h & \mathbf{B}_h & \mathbf{C}_h \\ \mathbf{A}_\theta & \mathbf{B}_\theta & \mathbf{C}_\theta \end{pmatrix} \begin{pmatrix} V \\ H \\ M \end{pmatrix} = \begin{pmatrix} \rho_v \\ \rho_h \\ \theta \end{pmatrix} \quad (9)$$

where the coefficients of the sub-matrices are as follows

$\mathbf{A}_{vij} = \rho_{a1}\alpha_{ij}\cos^2\psi_j + \rho_{NQ1}\alpha_{\rho Hij}\sin^2\psi_j$
$\mathbf{B}_{vij} = \rho_{a1}\alpha_{ij}\cos\psi_j\sin\psi_j - \rho_{NQ1}\alpha_{\rho Hij}\sin\psi_j\cos\psi_j$
$\mathbf{C}_{vij} = -\rho_{NM1}\alpha_{\rho Mij}\sin\psi_j$
$\mathbf{A}_{hij} = \rho_{a1}\alpha_{ij}\sin\psi_j\cos\psi_j - \rho_{NQ1}\alpha_{\rho Hij}\cos\psi_j\sin\psi_j$
$\mathbf{B}_{hij} = \rho_{a1}\alpha_{ij}\sin^2\psi_j + \rho_{NQ1}\alpha_{\rho Hij}\cos^2\psi_j$
$\mathbf{C}_{hij} = \rho_{NM1}\alpha_{\rho Mij}\cos\psi_j$
$\mathbf{A}_{\theta ij} = -\theta_{N1}\alpha_{\theta Hij}\sin\psi_j$
$\mathbf{B}_{\theta ij} = \theta_{N1}\alpha_{\theta Hij}\cos\psi_j$
$\mathbf{C}_{\theta ij} = \theta_{M1}\alpha_{\theta Mij}$
ρ_{a1} axial deflection of single pile due to unit axial load
ρ_{NQ1} normal deflection of single pile due to unit normal load
ρ_{NM1} normal deflection of single pile due to unit moment
θ_{N1} rotation of single pile due to unit normal load
θ_{M1} rotation of single pile due to unit moment

The above unit deflections and rotations may be calculated from the theoretical relationships (Poulos (1971a); Poulos (1971b) and Poulos and Mattes (1971)) if values of the soil moduli can be estimated (Poulos (1973) and Poulos (1974)), or if pile load test data is available, from the pile deflections at the working loads. The interaction factors α are given in Poulos and Mattes (1971) while values of the interaction factors $\alpha_{\rho H}$, $\alpha_{\rho M}$, $\alpha_{\theta M}$ are given in Poulos (1971b).

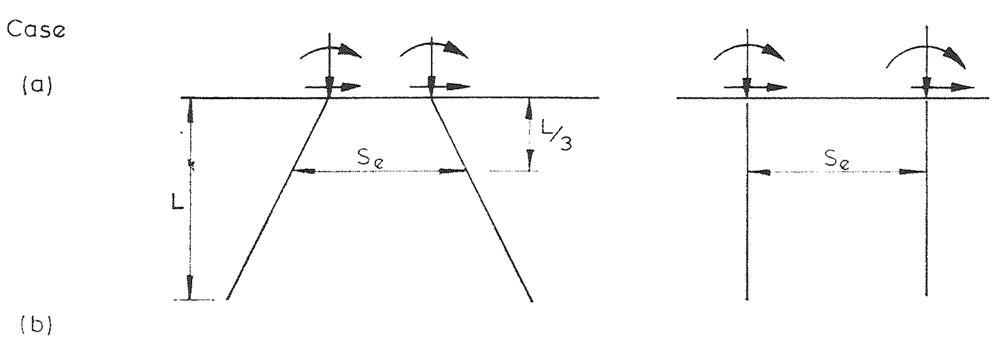
The submatrices $\mathbf{A}_v$, $\mathbf{B}_v$, etc., are of order $n \times n$ while the vectors, V, ρ_v etc., are of order n. Equation (9) together with the three equations expressing vertical and horizontal load equilibrium and moment equilibrium, may be solved to obtain the $3n + 3$ unknown vertical and horizontal loads, moments, displacements and rotations, for the desired boundary conditions at the pile heads.

A number of cases may be considered, including

(i) a rigid pile cap rigidly attached to the piles, so that the rotations and horizontal displacement of all piles are equal and the vertical displacement of a pile is related to its position in the group and the rotation.
(ii) piles pinned to a rigid pile cap, which is similar to case (i) except that the pile head moments are zero.
(iii) piles attached to a massive cap in which case horizontal and vertical displacements are equal but all pile head rotations are zero.
(iv) piles attached to a relatively flexible pile cap so that each pile is subjected to known loads and moments.

No account is taken in the above analysis of the horizontal shear and rotational resistance between the cap and the soil although the analysis could be extended to take these into account. Groups in which piles are battered in different directions can be treated approximately by resolving the horizontal load into two components and calculating the in-line horizontal displacements due to each component, using, as the length of a pile, its projected length in the plane of loading. The resultant horizontal displacement can then be calculated from these displacement components.

It should be emphasized that the fact that $3n + 3$ equations are required, rather than only 3 as in many methods based on subgrade reaction theory, is a consequence of considering the interaction between the piles rather than assuming that the displacements and rotation of a pile are a function only of the loads and moments on that particular pile. The consideration of inter-pile interaction in a logical manner in the present analysis thus obviates the necessity to make approximate allowances for group effects as in the equivalent bent approach. To evaluate (9) a computer program GUGEL (Groups Under GEneral Loading) has been written.

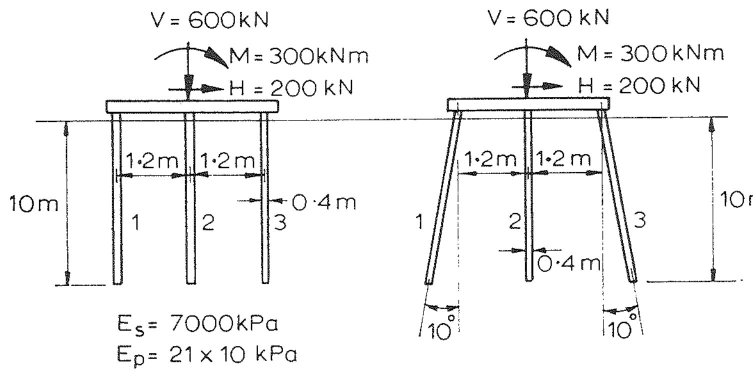


Two battered piles *Vertical piles at equivalent spacing*

Fig. 4—Equivalent spacing of battered piles.

COMPARISON BETWEEN METHODS OF PILE GROUP ANALYSIS

To compare the three methods of analysis described in this paper, two simple planar pile groups have been analysed, as shown in Fig. 5. Each group has three piles, and in the first, all piles are vertical while in the second, the outer piles are battered. In applying the equivalent bent method, the equivalent length of each member has been taken as the mean of L_{eH} and L_{eM} (Table I). A computer program (Harrison, 1973) has been used to evaluate the solution. For the elastic analysis, the single pile vertical and horizontal responses and the interaction factors have been obtained from the theoretical solutions in Poulos (1971a); Poulos (1971b) and Poulos and Mattes (1971).



Group A *Group B*

Fig. 5—Pile groups considered in comparison of methods.

The piles are assumed to be rigidly attached to a rigid cap in both cases. The loads, moments and deflections from each method of analysis are summarised in Table III.

The main points of interest are

(i) the vertical pile loads from the three methods agree quite closely, although the elastic analysis tends to predict a higher maximum load.

(ii) there is a considerable discrepancy between the computed pile moments from the equivalent bent and elastic analysis. The simple statical analysis assumes zero moment in all piles.

(iii) the equivalent bent approach predicts a smaller rotation than the elastic analysis and a larger vertical deflection of the leading pile, but a smaller horizontal deflection.

TABLE III
Comparison of methods of group analysis

	Quantity		Simple statical analysis	Equivalent bent analysis	Elastic analysis
Group A	V_1	kN	75	67.2	45.9
	V_2	kN	200	200.0	169.6
	V_3	kN	325	332.8	384.5
	H_1	kN	66.7	66.6	74.9
	H_2	kN	66.7	66.7	50.2
	H_3	kN	66.7	66.6	74.9
	M_1	kN m	0	−6.2	−39.6
	M_2	kN m	0	−6.2	−27.1
	M_3	kN m	0	−6.2	−39.6
	ρ_{v1}	mm	—	17.5	14.6
	ρ_h	mm	—	8.9	11.8
	θ		—	.00581	.00683
Group B	V_1	kN	75	59.3	62.6
	V_2	kN	200	200.3	169.1
	V_3	kN	325	329.6	368.3
	H_1	kN	38.8	76.7	56.6
	H_2	kN	52.0	75.5	42.1
	H_3	kN	109.2	47.8	101.3
	M_1	kN m	0	−43.3	−36.0
	M_2	kN m	0	−26.9	−21.3
	M_3	kN m	0	+66.9	−10.0
	ρ_{v1}	mm	—	16.4	13.9
	ρ_h	mm	—	8.2	9.5
	θ		—	.00490	.00540

It should be noted that the computed rotation and horizontal deflection in the equivalent bent method are sensitive to the equivalent length of the piles. For example, for Group A, if the equivalent length was taken as L_{eM} (= 1.96 m) instead of the mean of L_{eM} and L_{eH} (= 2.24 m), the vertical deflection and rotation are 16.8 mm, 6.7 mm and .00521 compared with 17.5 mm, 8.9 mm and .00581 in Table III. On the other hand, if L_{eH} (= 2.52 m) is used, the corresponding values are 18.2 mm, 11.4 mm and .00639. The latter values of horizontal deflection corresponds more closely to that from the elastic analysis and hence the use of an equivalent pile length equal to L_{eH} appears desirable.

A more detailed comparison of the computed deflection and rotation under the individual components of load reveals that the vertical movement due to vertical load given by the equivalent bent method and elastic agree closely but that the computed rotation due to both horizontal load and moment is considerably smaller in the equivalent bent method. The equivalent bent method also gives a larger horizontal deflection due to moment but a smaller horizontal deflection due to horizontal load.

The above comparisons therefore highlight the difficulty of attempting to characterise a complex pile-soil system by a structural frame. Because it is a more rational nature, the elastic analysis should give more reliable deflection predictions and is recommended. However, in cases where the piles in the group have a significant unsupported length above the ground line or where the pile cap cannot be considered as rigid, the equivalent bent approach may provide a more convenient means of analysis.

PARAMETRIC STUDIES OF TYPICAL PILE GROUPS

In this Section, solutions for the displacement and rotation of some typical pile groups are presented. The effects of the following factors on group behaviour are examined: pile stiffness, pile batter, pile spacing and pile configuration. The results are expressed in terms of dimensionless influence factors and have been obtained from the elastic interaction analysis.

Effect of pile stiffness and batter angle

The effects of pile stiffness and batter angle on the deflection and rotation of a pile group are illustrated in Fig. 7 for a group of six 25 diameter piles in a deep soil layer, as shown in Fig. 6, for $s = 3d$. The piles are assumed to be rigidly attached to a rigid pile cap, and the soil is elastic and has a Young's modulus which is constant with depth. Three values of pile stiffness factor K are considered, $K = 100$ (corresponding to concrete piles in a stiff soil), $K = 1\,000$ (corresponding to concrete piles in a medium-stiff soil) and $K = 10\,000$ (corresponding to concrete piles in a soft soil). For each value of K, the value of pile flexibility factor K_R is related as follows

$$K_R = \frac{K I_p}{R_A L^4} \tag{10}$$

where I_p moment of inertia of pile section

R_A area ratio, defined in equation

L pile length

The vertical and horizontal deflections, ρ_v and ρ_h, and the rotation θ are expressed as follows

$$\rho_v = \frac{V}{E_s d} I_{vv} + \frac{H}{E_s d} I_{vH} + \frac{M}{E_s d^2} I_{vM} \tag{11}$$

$$\rho_h = \frac{V}{E_s d} I_{Hv} + \frac{H}{E_s d} I_{HH} + \frac{M}{E_s d^3} I_{HM} \tag{12}$$

$$\theta = \frac{V}{E_s d^2} I_{\theta v} + \frac{H}{E_s d^2} I_{\theta H} + \frac{M}{E_s d^3} I_{\theta M} \tag{13}$$

where V vertical load on group

H horizontal load on group

M moment on group

I_{vv}, I_{vH} etc. are dimensionless deflection and rotation coefficients evaluated from the analysis

E_s Young's modulus of soil

For the symmetrical group considered here, $I_{HV} = I_{\theta V} = 0$, i.e. the horizontal deflection and rotation due to unit vertical load are zero.

The coefficients generally decrease (i.e. the deflections and rotation decrease) as the batter angle of the piles increases. However, the factors primarily influenced are the vertical deflection and rotation due to horizontal load (I_{VH} and $I_{\theta H}$), and the horizontal deflection due to horizontal load and moment (I_{HH} and I_{HM}). The other coefficients are virtually unaffected by pile batter. The pile stiffness factor K has a significant effect on most coefficients.

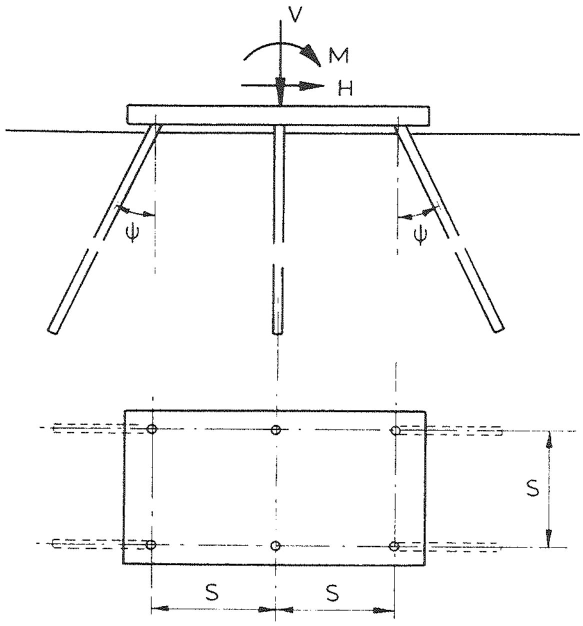


Fig. 6—Pile group considered in parametric study.

Effect of pile spacing

The effect of pile spacing on the deflection and rotation coefficients for a 6-pile group is shown in Fig. 8 for a batter angle of 15°. Almost all coefficients decrease with increasing spacing, as would be anticipated. The decrease is generally more marked for lower values of K.

Effect of pile configuration

In order to examine the effect of pile configuration on group rotations and deflections, the six groups shown in Fig. 9 have been analysed. Group A is the one shown in Fig. 6 while Group B is the same group except that the centre two piles are removed. The other four groups have different piles battered. In all cases, the batter angle of any battered piles is 15°.

The deflection coefficients for each group are shown in Table IV. The following observations may be made

(i) The behaviour of Group A is very similar to that of Group B, i.e. the centre piles in Group A have little influence on the deflection and rotation coefficients.

(ii) The advantages of Group D over Group C arise primarily from the negative horizontal deflection and rotation developed under vertical load.

(iii) Groups E and F behave similarly i.e. battering the centre piles has little influence on the group deflection.

TABLE IV

Effect of pile configuration on deflection and rotation coefficients*
(see Fig. 9 for details of pile groups)

Coefficient \ Group	A	B	C	D	E	F
I_{VV}	.0528	.0566	.0613	.0542	.0525	.0530
I_{VH}	.00627	.00445	.0258	.00300	.00311	.00152
I_{VM}	.00491	.00506	.00651	.00484	.00504	.00509
I_{HV}	0	0	.0150	−.0150	−.00510	−.00810
I_{HH}	.0945	.0981	.1089	.1091	.1026	.1021
I_{HM}	.00036	−.00023	.00378	.00378	.00186	.00188
$I_{\theta V}$	0	0	.00116	−.00116	−.00039	−.00045
$I_{\theta H}$	.00209	.00148	.00378	.00378	.00286	.00286
$I_{\theta M}$	.00164	.00169	.00188	.00188	.00171	.00171

*Coefficients are for the leading piles of the group.

In order to gain a better appreciation of the relative merits of the six groups considered, a numerical example has been taken in which $L = 10$ m, $d = 0.4$ m, $E_s = 7000$ kN/m², $V = 1200$ kN, $H = 400$ kN and $M = 600$ kNm. The resulting deflections and rotations, calculated from (11) to (13) and the coefficients in Table IV, are shown in Table V. It is evident that Group C is less satisfactory than the others. Overall, Groups E and F deflect the least, although Group B is little inferior to these Groups or to Group A, and would be preferred from the point of view of economy, provided that vertical and lateral stability is adequate.

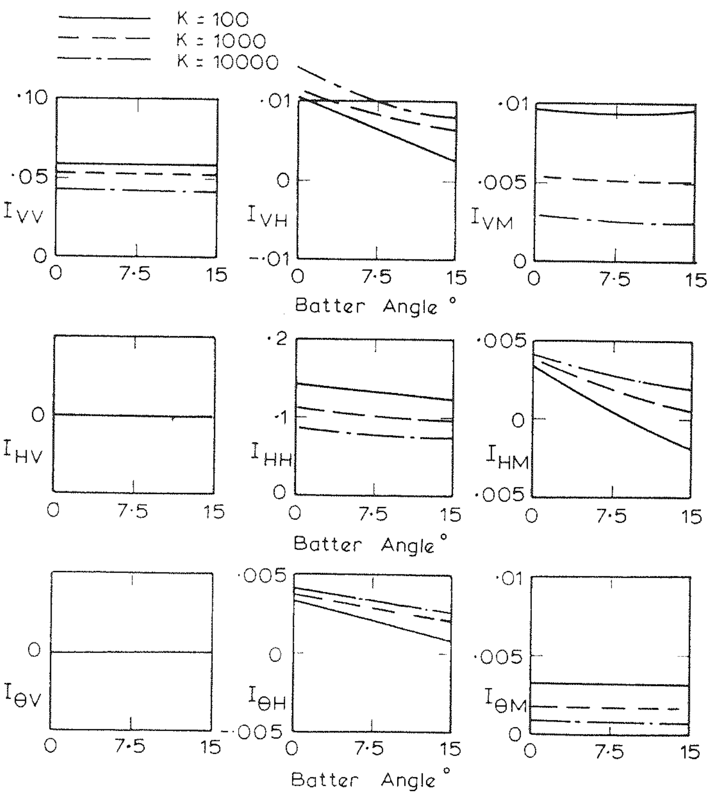


Fig. 7—Effect of batter angle and relative pile stiffness on deflection and rotation coefficients.
(6-pile group, $L/d = 25$, $\nu_s = 0.5$, $s/d = 3$.)

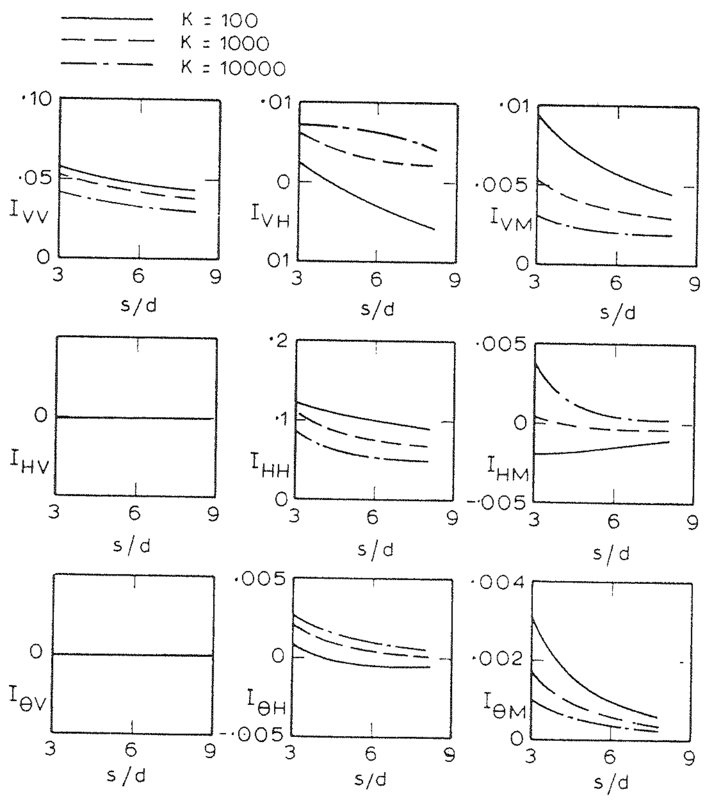


Fig. 8—Effect of pile spacing on deflection and rotation coefficients.
(6-pile group, $L/d = 25$, $\nu_s = 0.5$, batter angle $\psi = 15°$.)

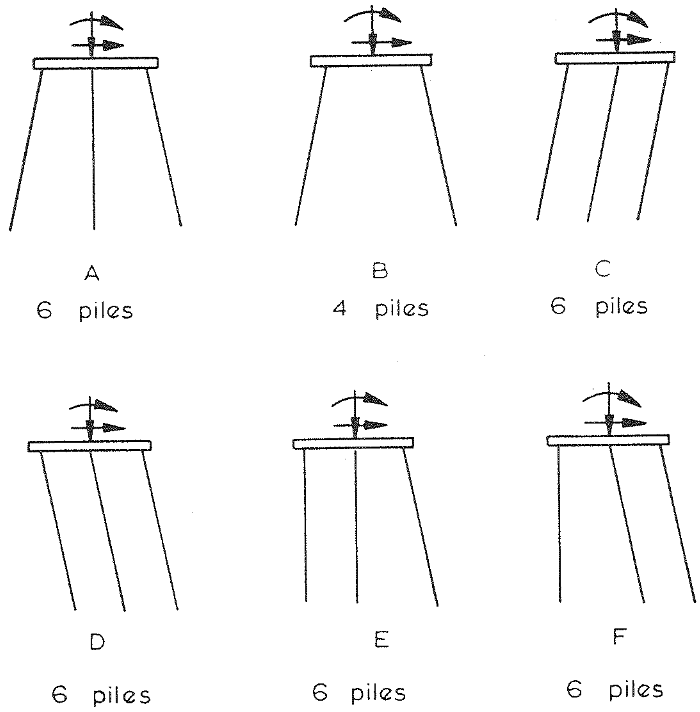


Fig. 9—*Groups considered in parametric study of effect of pile configuration.*
(In all cases, pile spacing at cap = 3*d*,
$L/d = 25$, $K = 1000$, $\nu_s = 0.5$, *batter angle* $\psi = 15°$.)

TABLE V

Comparison of group performance

Group / Quantity	A	B	C	D	E	F
ρ_v mm	26.2	27.6	33.5	26.3	25.6	25.7
ρ_h mm	13.7	13.9	24.0	11.2	13.5	12.1
θ	.00294	.00279	.01727	.01478	.00289	.00283

$V = 1200$ kN, $H = 400$ kN, $M = 600$ kNm
$L = 10$ m, $d = 0.4$ m, $E_s = 7000$ kN/m^2

CONCLUSIONS

Comparisons between three approaches to the analysis of pile groups under general loading systems have shown that similar vertical loads in the piles are calculated by the three methods, but that horizontal loads and moments may differ considerably, even to the extent of differing in sign. These differences arise from the manner in which inter-pile interaction is considered; the largest loads and moments appear to be predicted from the elastic interaction analysis which takes the most logical account of interaction, while the smallest loads are predicted by the simple statical analysis, in which no account of interaction is taken. Despite considerable differences in computed loads and moments, the deflections and rotations given by the equivalent bent method and the elastic interaction analysis are similar, although differences do exist in certain components of the horizontal deflection and rotation. The elastic interaction analysis appears to be the most logical method but in cases where the piles in the group have a significant unsupported length above the ground line or where the pile cap has finite flexibility, the equivalent bent approach may be easier to apply.

Parametric studies of typical pile groups have shown that the components of deflection and rotation most influenced by pile batter are the vertical deflection due to horizontal load, the horizontal deflection due to moment and the rotation due to horizontal load. The pile stiffness most influences the deflections and rotation due to moment. As would be expected, increasing the spacing between the piles leads to significant decreases in most of the components of deflection and rotation. Finally, the solutions indicate that the centre piles in a six-pile group serve little useful purpose as far as deflections and rotations are concerned.

The elastic interaction analysis described in this paper assumes linear response of the piles to load, but it is possible to introduce non-linearity by carrying out an iterative analysis in which the unit deflections and rotations are dependent on load level. A theoretical method of assessing this dependence is described by Poulos (1972, 1973). Calculations have shown that the interaction factors may reasonably be assumed to be independent of load level.

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