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SOIL-STRUCTURE INTERACTION

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1 INTRODUCTION

The term "soil-structure interaction" can refer to a wide range of topics, including the behaviour of buried pipes, tunnel linings and conduits, retaining structures as well as the interaction between a building and its foundation. In this article, the scope will be restricted to foundations for buildings that may consist of surface raft foundations or deep foundations such as piled raft foundations.

Because structures have been traditionally designed by structural engineers and foundations by geotechnical engineers, there has tended to be a separation of design work into that of the structure and that of the foundation. Either the stiffness of the structure is neglected and uniform loads are applied to the soil surface, or the structure is analysed alone and the soil is treated as a series of springs representing the foundation soil. In the case of a piled raft, the piles supporting the raft are often treated as springs. Neither approach is satisfactory, and some form of interaction analysis is warranted to take the structural stiffness into account.

Much of the pioneering work in this area was carried out in Australia by Fraser and Wardle (1976), Hain and Lee (1974, 1978), Poulos and Davis (1980) and Brown (1969a,b) to name a few. The development of methods for the analysis of soil-structure interaction involving foundations has developed a great deal since this early research and convenient accounts can be found in the books by Hemsley (1998, 2000). Solution methods range from simple hand calculation techniques to semi-analytical and numerical methods that require complex computer analysis. Each type of analysis has its field of application and, in general, simple techniques can be used in the early stages of design to obtain some understanding of foundation behaviour before more complex analysis is undertaken.

2 THE USE OF RAFT AND PILED RAFT FOUNDATIONS

Generally piled foundations can be used for tall structures founded on compressible materials where the piles can easily reach bedrock or stiff underlying layers. However, if the stiffer layers are deep, then raft or piled raft foundations become a feasible alternative.

A good example of this is Frankfurt, where the Frankfurt clay is between 30-50m deep and overlies the Frankfurt limestone. The original foundations for skyscrapers in Frankfurt were therefore raft foundations that were 2 to 4m thick. However these raft foundations tended to undergo fairly large settlements of between 15 to 35cm. An example is the Deutsche Bank building that settled about 19cm between 1978 and 1985.

As well as settling, these structures experienced a tilt (1/870 for the Deutsche bank) and this caused serviceability problems, because a small tilt can mean a large movement at the top of buildings that are 200 to 250m tall.

This led to the use of piled raft foundations in Frankfurt. This type of foundation transfers load to the subsoil through the surface raft as well as through the piles, and has proven to be a popular means of founding tall buildings in this city. One of the first structures to be built using a piled raft was the Messeturm, a 256m high structure founded on a 3-6m thick raft and supported by 64 piles of average length 30m and 1.3m in diameter. This structure did not tilt and the settlement was about 14cm (Katzenbach et al, 2001).

Today many structures around the world are constructed on piled raft foundations, for example the Emirates Twin Towers in Dubai (Figure 1) and some of the buildings in Kuala Lumpur. Different methods of analysis have been developed over the years for analysis of raft and piled raft foundations and in the following these methods are reviewed and used for analysis of case studies.

Figure 1: The Emirates twin towers Dubai, constructed on piled raft foundations (courtesy Hyder Consulting, Australia)

3 SURFACE RAFT FOUNDATIONS

For raft or mat foundations constructed on or slightly below the soil surface, many different analytic approaches have been developed and these are listed in the following subsections.

3.1 SPRING OR WINKLER FOUNDATIONS

One of the earliest and simplest approaches to soil-structure interaction was to represent the resistance of the soil by a spring. This approach was applied to laterally loaded piles as well as raft foundations. However, for raft foundations, this approach can lead to serious error because:-

(1) The springs are independent and do not interact. Therefore the compression of one spring does not influence other parts of the foundation. To illustrate this, consider the case of a uniformly loaded raft. Such a raft will undergo a uniform displacement and therefore there will be no bending moment predicted in the raft. This is obviously wrong, as it is observed that such a loading would make a rectangular raft (for example) deform into a dished shape, and the raft would then carry bending moments.

(2) It is difficult to establish the stiffness values for the springs that are used in analysis because the spring constants are dependent on the scale of the foundation. For example, if a modulus of subgrade reaction is determined from a plate loading test, the load-deflection behaviour is specific to the size of plate used in that test. It should not be applied to loaded areas that are different in size to that of the plate.

(3) A Winkler or spring model cannot directly take account of soil layering.

(4) A vertical loading on a foundation may cause lateral displacements. A spring model cannot be used for such predictions.

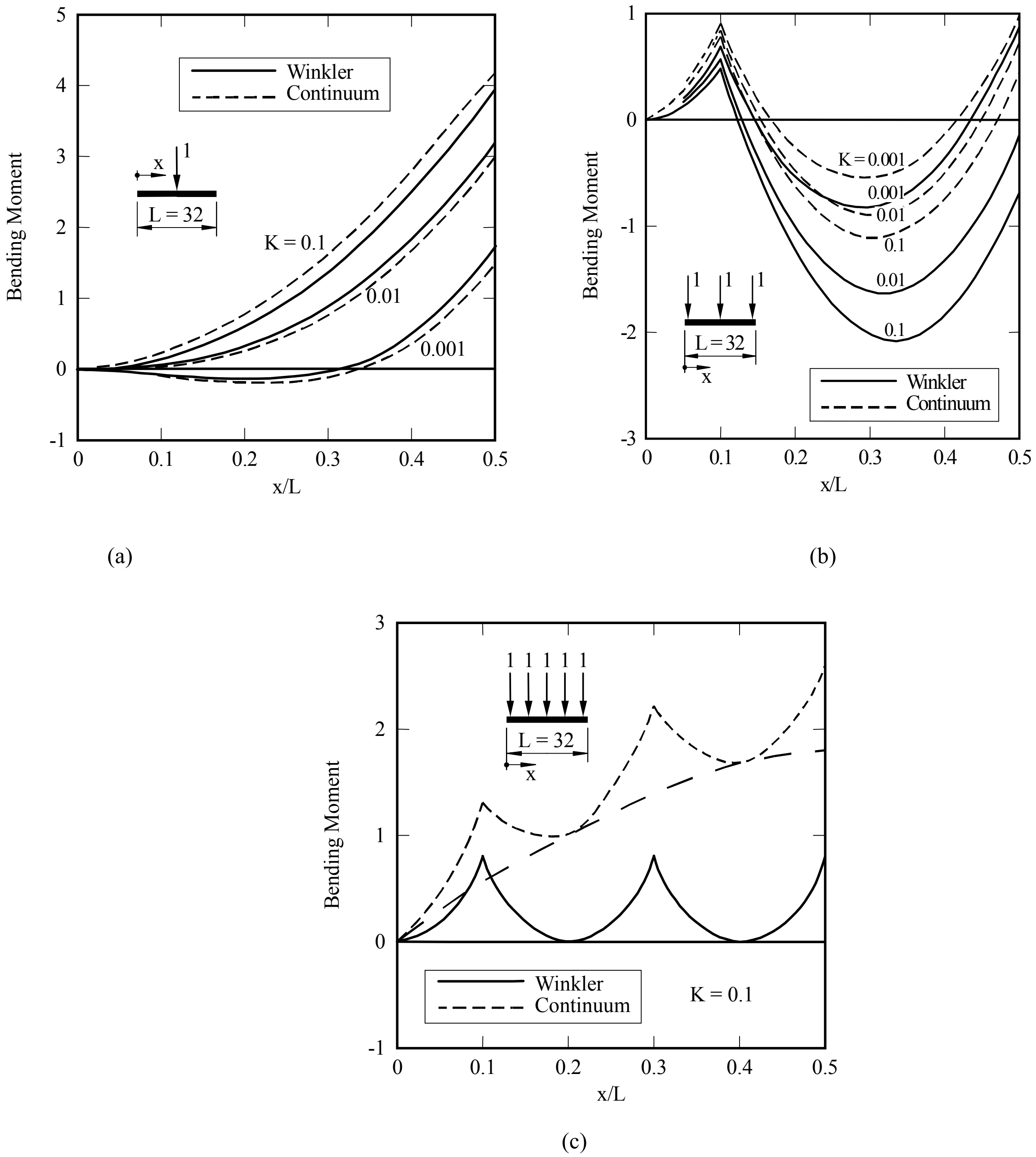


Figure 2: Moments in strip raft – Winkler and continuum solutions

Because of the limitations listed above, it is desirable to use continuum models for the soil (i.e. treat it as being an elastic or elasto-plastic material). An example of the differences in solutions obtained by using a spring model and a continuum model has been presented by Brown (1977) to illustrate the difference in the choice of soil model. The problem involves unit point loads applied to a strip raft (L/B = 10). In order to compare the two models, the modulus of elasticity of the soil (continuum model) and of the subgrade reaction (spring model) were chosen so that the settlements of a rigid strip foundation with a central point load are equal. Figures 2a,b,c show the computed moments in the raft where the raft stiffness is defined as K

$$K = \frac{16EI(1-\nu_s^{\,2})}{\pi E_s L^4}$$

and where

EI = bending stiffness of the raft
ν_s = Poisson's ratio of the soil
L = length of the raft
B = width of the raft
E_s = Young's modulus of the soil

From the figures, it may be seen that the calculated moments in the raft show reasonable agreement for the central point load only. For the multiple point load cases there is a large difference in the calculated moments. As the load becomes more uniformly distributed, the relative errors generally increase.

It may therefore be concluded that the use of spring models may lead to large errors and should not be used for raft foundation design, particularly in the case of distributed loading.

3.2 SOLUTIONS FOR UNIFORM CONTINUA

Solutions that treat the soil as an elastic continuum are superior to spring models in that they allow for interaction between loaded portions of a foundation or adjacent foundations. Early solutions to the problem of a foundation on an elastic continuum involved assuming that the contact stress between the raft and the foundation could be approximated either as a series of blocks of uniform pressure (Zhemochkin and Sinitsyn, 1962) or as an arithmetic series (Gorbunov-Possadov and Serebjanyi, 1961).

For vertical loading the deflection of the soil at the interface can then be assumed equal to the deflection of the raft at selected locations, and enough equations can be established to solve for the magnitudes of the blocks of pressure or the unknown coefficients of the terms in the arithmetic series.

Solutions obtained using these approaches include those of Cheung and Nag (1968), and Cheung and Zienkiewicz (1965) for structures on infinitely deep soils, Brown (1975a) for strip footings carrying point loads and for circular raft foundations (Brown 1969a,b) on elastic soils of infinite or finite depth. Selvadurai (1980) and Rajapakse (1988) have also presented solutions for circular rafts on infinitely deep soils.

In most of these analyses, the raft was analysed by treating it as a plate or thin shell, so the theory of Timoshenko and Woinowski-Krieger (1959) could be used. Whether the use of thin shell theory is justified when real foundations can be several metres thick has been examined by the writer, and it has been found that this assumption is reasonable for most cases, as it is the thickness to width ratio of the foundation that is generally most important.

3.3 METHODS FOR RAFTS ON LAYERED SOILS

For soils that are horizontally layered so that the properties of the layer do not vary in the horizontal direction, either Fourier series or Fourier transforms may be applied to the field variables (i.e. displacements and stresses) to obtain solutions. Transforms can be applied to the contact stress represented either as uniform blocks of pressure or as an arithmetic function. Blocks of pressure may be used more generally, as they can be used with any shape of raft foundation and any loading pattern. Arithmetic functions can only be used in certain cases, for example a circular raft with a uniform load, where the form of the functions can be chosen to suit the problem.

An early solution to this type of problem was obtained by Fraser and Wardle (1976) who used integral transform techniques (Fourier transforms) to obtain the response of the soil to the contact stress applied by the raft. The raft was analysed using finite element techniques. They presented solutions for the settlement and bending moment in uniformly loaded rectangular rafts on layered soils of finite thickness. Their method of computing the behaviour of the layered soil was approximate, involving a weighting of the elastic parameters of each layer to obtain an 'average' set of parameters.

Tham et al (1988) first used finite layer methods to obtain a more rigorous solution to the problem of a raft on a layered soil. Zhang and Small (1992) also demonstrated the use of finite layer methods to analyse a raft on a layered soil. Fourier transforms were used to obtain the response of the soil to blocks of uniform pressure, and finite element analysis was used for the raft. This approach allowed a rigorous analysis of rafts on layered anisotropic soils of finite depth and could easily incorporate lift-off of the foundation or could be used to limit the contact stress to a maximum value in order to approximately model soil yield.

These semi-analytical approaches have appeal in that they may be used to analyse what is essentially a three dimensional problem, with fairly simple data input and much smaller sets of equations. Their limitation is that they only approximately deal with soil yield, and soil layers must be horizontal, continuous and uniform. However, in most practical cases these limitations are not of great significance as, in general, loading is well below the failure load of the soil and sedimentary soils are often layered horizontally.

An example of a solution to the problem involving a raft on a layered soil is shown in Figure 3. The program FEAR (Finite Element Analysis of Rafts, Small 2002), based on finite layer theory, was used to compute the results. With this technique, the contact stress beneath the raft is treated as a series of uniform blocks of pressure that correspond to each element in the raft. The deflection of a layered soil can then be calculated for each of the rectangular blocks of uniform pressure using the finite layer technique (Small and Booker, 1986). The method is very simple to use, as the raft can be of any shape and can carry point, uniform or point moment loadings and (for the soil) only the thickness of each soil layer and its elastic properties are required.

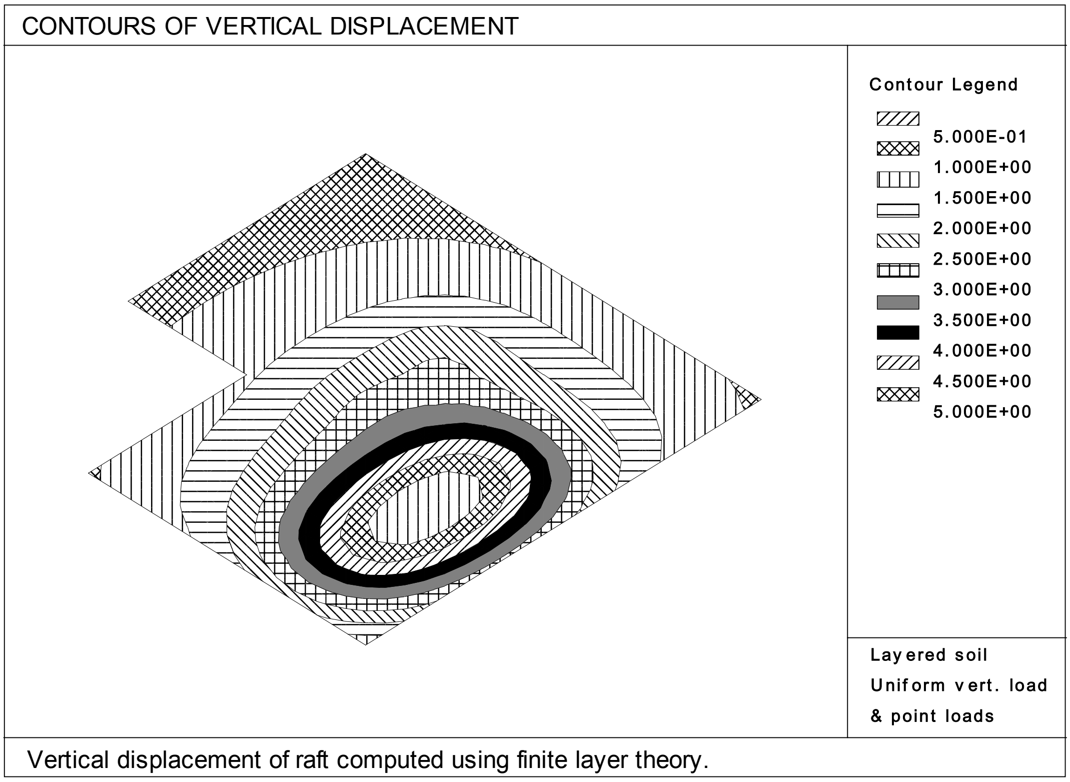


Figure 3: Example of graphical output from finite layer analysis (deflections in mm)

3.4 FINITE ELEMENT METHODS

The most powerful method for analysing rafts is the finite element method. A full three-dimensional mesh can be developed and the raft and complete structure on top of the raft can be incorporated. Different constitutive laws can be used for the soil such as advanced elasto-plastic models.

An early demonstration of the technique was made by Smith (1970) but for an axisymmetric problem only, while Cheung and Zienkiewicz (1965) looked at general shaped rafts. With the development of computers, it is now possible to analyse quite complex three-dimensional problems by using desk-top computers (e.g. Milovic, 1998).

3.5 EFFECT OF SUPERSTRUCTURE

In most of the analyses reported in the previous sections, only the raft foundation has been considered in the analysis. The actual superstructure is not considered, and column loads or distributed loads and moments are applied directly to the foundation.

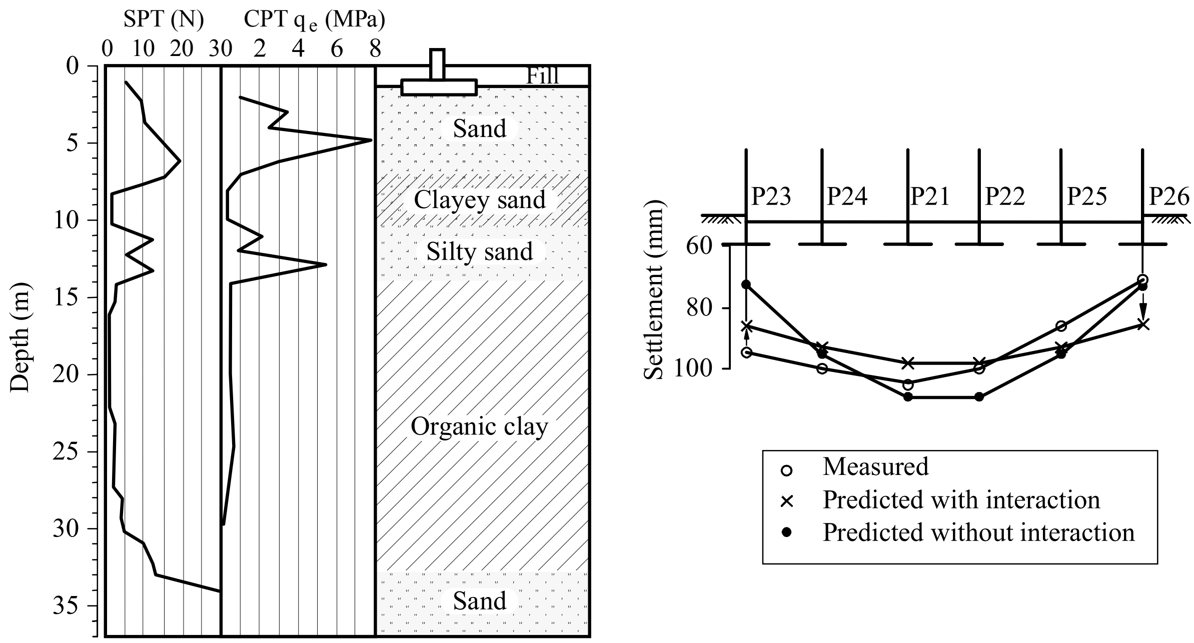


Figure 4: Deformation of foundation calculated with and without the superstructure

If there are extremely stiff structural elements such as shear walls or solid cores (i.e as used for lift shafts), these can be approximated by using very stiff elements for the raft. These elements will then behave in a very rigid fashion, attracting moment and allowing little differential deflection in the region of the stiff element.

Several researchers have examined the effects of superstructures that do not behave as being very rigid; and these include Lee and Brown (1972), Hain and Lee (1974), Lee (1975), Poulos (1975) and Brown and Yu (1986).

Fraser and Wardle (1975) presented results for a 2 bay portal frame where they showed that the differential deflections in the frame depended on the stiffness of the frame. Brown (1975b) has also shown, for a strip raft beneath a 2-dimensional frame, that the relative stiffness of the structure has an effect on differential displacement in the raft. Zhang and Small (1994) analysed 3-dimensional framed buildings on raft foundations and demonstrated that the larger the relative stiffness of the building frame, the smaller the differential deflections in the raft.

Brown and Yu (1986) also showed that, as a building is constructed, the stiffness of the overall structure increases and this affects the differential displacement in the raft. Gusmau Filho and Guimaraes (1997) have also looked at construction sequence and have noted that the loads in columns reach a maximum (or minimum) value as more storeys are added to the building, leading to the idea of the building reaching a "limit stiffness".

An example showing how incorporation of the stiffness of the structure into the analysis can improve the predicted behaviour of a foundation has been presented by Lopes and Gusmao (1991). For a 15 storey structure in Brazil,supported by a system of strip footings, the settlement distribution was shown to be predicted more closely if the stiffness of the structure is included in the settlement analysis (see Figure 4).

Most of the papers cited in this section have contained the conclusion that the stiffness of the structure does have an effect on the differential deflection in the raft foundation, although for flexible framed structures the effect is small (see Yao and Zhang, 1985).

4 PILED RAFT FOUNDATIONS

If a surface foundation is not adequate to carry structural loads without excessive differential deflections, piles may be needed. Both the raft and the piles then transfer load to the soil and the interaction problem involves both the raft and the piles. In some cases, the piles are only placed beneath the raft to provide differential settlement control and are allowed to reach a high percentage of their maximum load capacity (Hansbo and Kallestrom, 1983).

It is important to realise that piles do not need to be uniformly placed over a foundation, but can be judiciously placed so as to carry the larger loads or to limit the differential deflections. In this regard, it is useful to have a quick and simple computer program or simple design method that can be used in the design stage to determine the best layout of the piles beneath the foundation. For example, Horikoshi and Randolph (1997) have shown that the optimum design of a piled raft carrying a uniform load would involve piles placed under the central 16 to 25% of the raft area.

Initially, piles were treated as groups that were either rigidly joined at the head or carried equal loads and the flexibility of the raft that joined the pile heads was ignored. The book by Poulos and Davis (1980) includes many of the methods

for computing the settlement of piles or pile groups when the raft is assumed to be totally rigid or totally flexible (i.e. raft flexibility is one of two extremes). These solutions are based on treating the shear forces acting down the pile shaft as a series of uniform shear stresses acting over sections of the pile shaft. Mindilin's equation for a subsurface point load is integrated over the section of pile to obtain the solution for the effect of the uniform shear stress on deflections of the soil at other sections of pile for the pile itself or for other piles. Interaction between piles can therefore be found using this technique often called a 'boundary element' technique.

Many different means of analysing piled raft foundations have been developed over the years (an excellent review has been provided by Randolph (1994). Some of the methods that can be used for piled rafts are similar to those used for surface rafts and so, once again, it is convenient to group them into the following classes:-

(1) Simple plate on springs approaches

These methods treat the piles as springs with the raft treated as a plate, and include the methods of Clancy and Randolph (1993), Poulos (1994) and Viggiani (1998).

(2) Boundary element methods

These methods employ the technique described above and include solutions obtained by Butterfield and Banerjee (1971), Brown and Weisner (1975), Hain and Lee (1978), Kuwabara (1989) and Chow (1986).

(3) Finite Layer techniques

Ta and Small (1996) used finite layer techniques to compute the behaviour of piled rafts, where the piles were driven into layered soils. Cheung et al (1988) had previously used series to analyse the behaviour of pile groups in layered soils, and the method can be extended to piled rafts. Zhang and Small (2000) have extended these techniques to horizontal loading of a piled raft.

(4) Simplified finite element or finite difference analyses

Analyses can be carried out by approximating the piles as a two dimensional or axi-symmetric body and assigning 'smeared' material properties to the piles in order to approximate the actual three dimensional behaviour. That is, the solid continuous 'pile' in an axi-symmetric or 2-d analysis is given a lower modulus to make it compress the same amount as the actual individual piles. Analyses of this sort include those of Desai et al. (1974) and Hooper (1973). Lin et al. (1999) have used a finite difference technique to compute the behaviour of the soil beneath a piled raft and applied the theory to piled rafts in Bagkok clay using a two-dimensional finite difference grid.

(5) Three-dimensional finite element analyses

As computer storage has increased, full 3-d analyses of piled rafts have been carried out and examples of this are given by Zhuang et al. (1991), Katzenbach and Reul (1997), Katzenbach et al. (1997), and Ottaviani (1975).

4.1 NUMERICAL MODELLING

In the previous section, many different methods of piled raft analysis were listed. The model chosen for a particular application would depend on the degree of sophistication required in the analysis. It is desirable to know the effects of assumptions made in the different types of analyses and so, in the following subsections, a limited examination is made of some aspects of the analyses listed.

4.1.1 *Finite layer techniques*

Simple techniques such as the finite layer methods of Zhang and Small (2000) that are used to compute soil movements from analytic or semi-analytic techniques, may produce errors because of the approximations made in the analysis. In order to test the accuracy of such methods, solutions were obtained from a three-dimensional finite element program, and from a finite layer program (APRAF) for a piled raft with a horizontally applied loading.

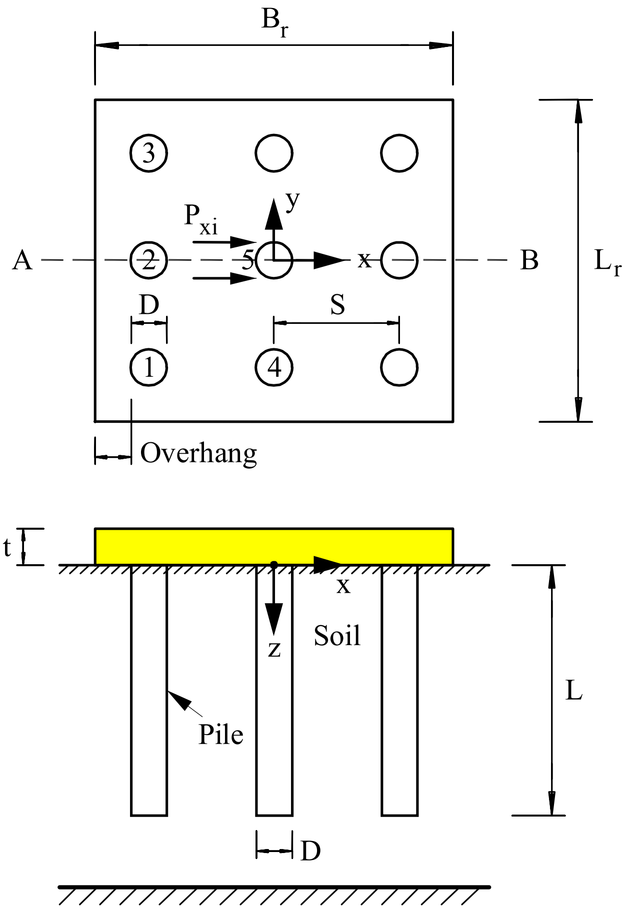


Figure 5: Layout of laterally loaded piled raft.

The raft is shown in Figure 5 and consists of a 3 by 3 pile group with a raft in contact with the ground surface. The raft overhangs the piles by one pile diameter (around the perimeter). The finite element mesh used to model this raft is shown in Figure 6 where it may be seen that one quarter of the raft is modelled because of symmetry. The mesh extends further in the x-direction because loading is to be applied to the raft in that direction, and the boundary should not affect the results by being too close.

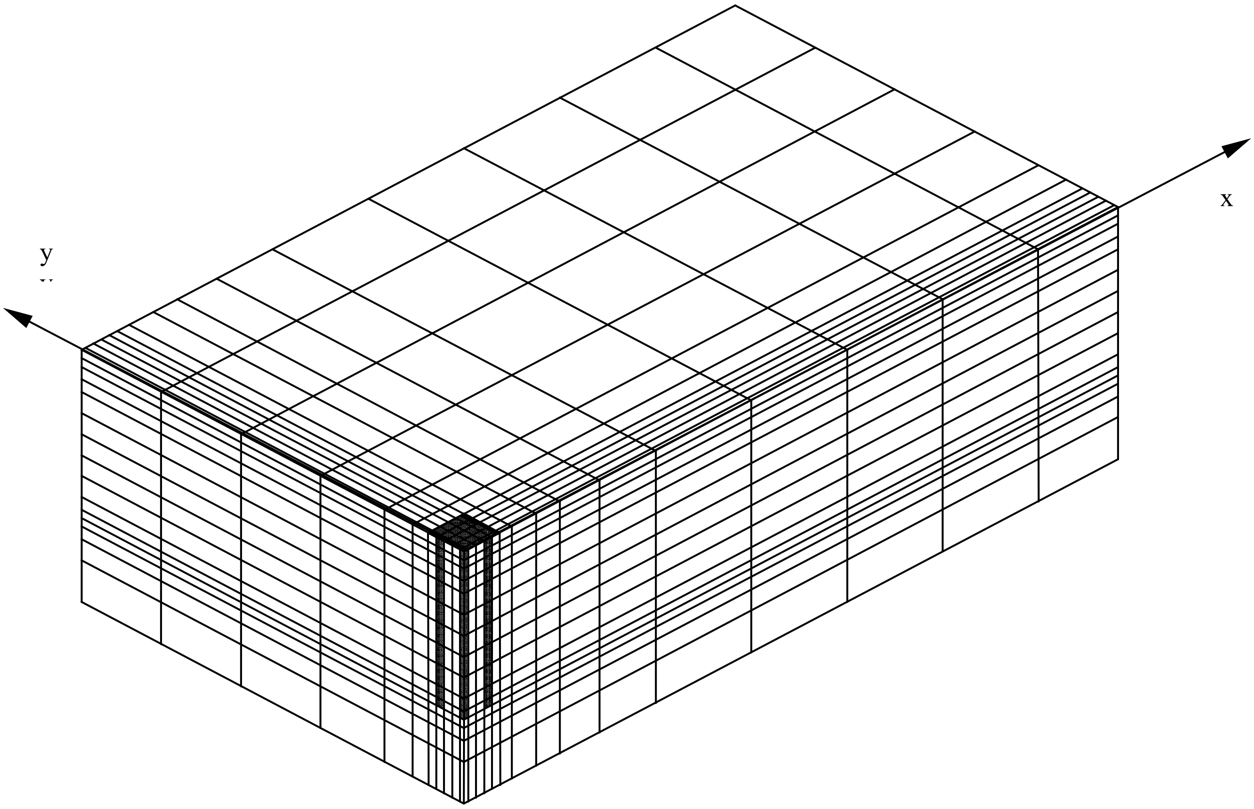


Figure 6: Finite element mesh used for lateral loading of piled raft problem.

All of the properties of the piled raft are given in Table 1.

Table 1: Properties of piled raft (3x3 group).

Quantity	Value
Pile diameter	0.5m
Pile length	10m
Depth of soil	15m
Raft width L_r	S/D = 3; 9m
Raft breadth B_r	S/D = 3; 9m
Overhang of raft	0.5m
Raft thickness	0.25m
Soil modulus	10MPa
Soil Poisson's ratio	0.3
Raft modulus	30,000MPa
Raft Poisson's ratio	0.3

Two horizontal point loads were applied to the heads of each pile (18 loads in all) making a total horizontal loading of 18MN. For the purposes of comparison, no slip was allowed between the raft and the soil, or the piles and the soil. The deflection of the raft can be calculated from the finite layer method, and a section (*A-B* in Figure 5) through the deformed raft is shown in Figure 7. In the figure it can be seen that the raft rotates under the horizontal loading and at its centre ($x = 0$) does not undergo vertical movement. The computed results from the finite layer and finite element methods can be seen to be in reasonably close agreement.

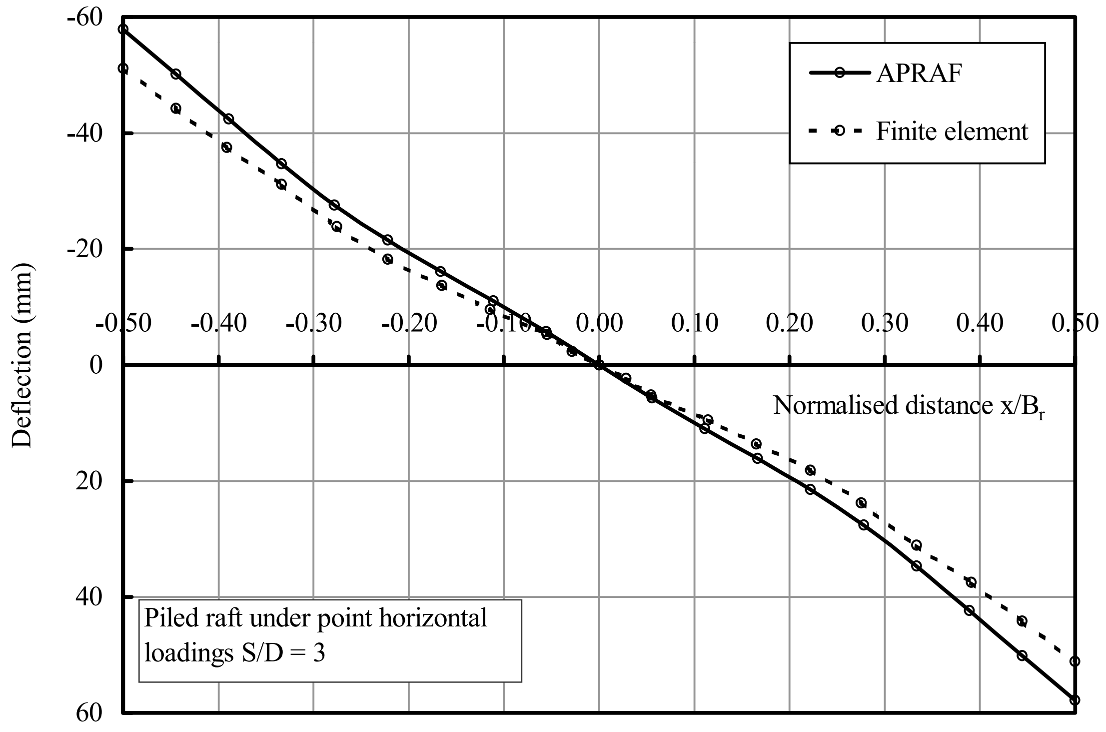


Figure 7: Vertical displacement of laterally loaded piled raft foundation along section A-B

The moments in the piles may also be computed, and are shown in Figure 8. In this figure, moments down the pile shaft are shown for pile 1 (the corner pile) and pile 5 (the centre pile), and it may be seen that there is very close agreement between the finite element and finite layer values.

4.2 APPLICATION

There is not a great deal of data involving the performance of foundations for structures. This is partly due to the fact that it is difficult to fully instrument foundations, and so often deflections at the foundation surface are all that are measured. Often the only data is for vertical loading, and not for horizontal loading, and this makes verifying theoretical computations for horizontal loading more difficult.

In order to gauge whether some of the numerical methods that have been mentioned can be applied with confidence in practice, the case of the Westend Street 1 Tower in Frankfurt Germany was examined. The building (shown in Figure 9) is 51 stories high (208 m) and has been described by Franke *et al*. (1994) and Franke (1991). The foundation for the building was a piled raft with 40 piles that were 30 m long as shown in Figure 9. The foundation was constructed in a deep deposit of the Frankfurt clay 120 m thick and, using pressuremeter tests reported by Franke *et al*. (1994), the average modulus of the clay was assessed to be 62.4 MPa.

The ultimate load capacity of each pile was computed to be 16 MN and a total load of 968 MN was assumed to be applied to the foundation (this is greater than the combined ultimate capacity of the individual piles).

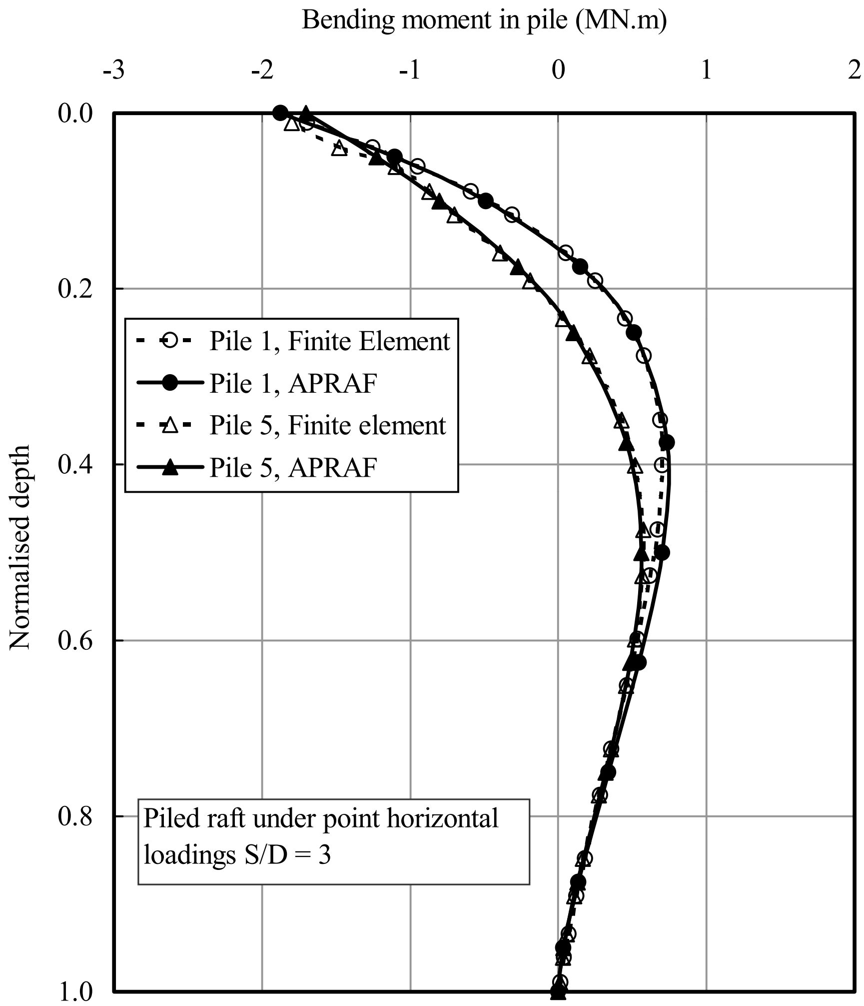


Figure 8: Moment variation with depth for piles beneath a laterally loaded piled shaft.

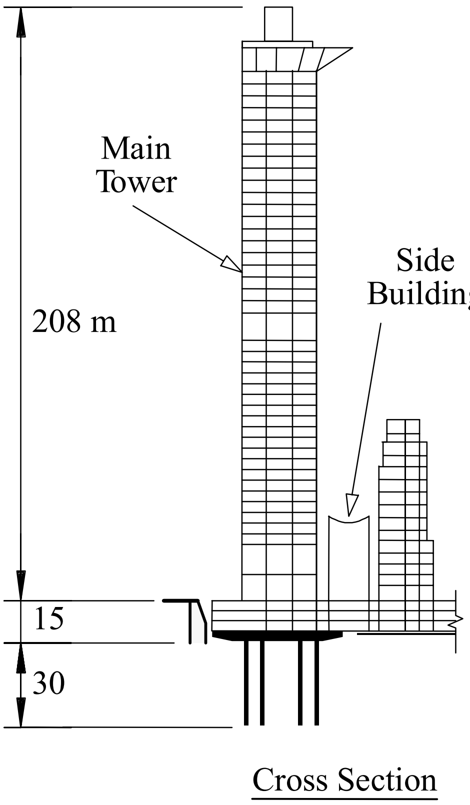
Six methods were used to predict the performance of the piled raft foundation:

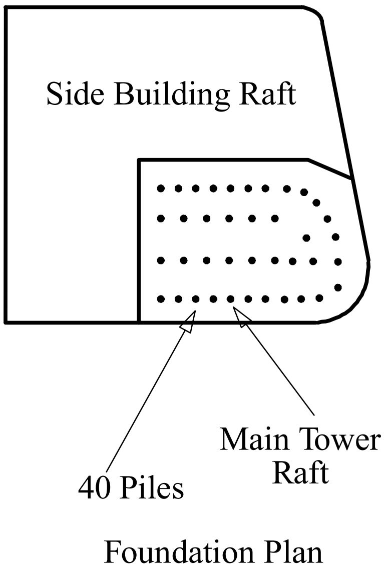
- The boundary element approach of Poulos and Davis (1980).
- Randolph's (1983) method.
- The strip on springs approach using the program GASP (Poulos, 1991).
- The raft on springs approach using the program GARP (Poulos, 1994a).
- The Finite element and finite layer method of Ta and Small (1996).
- The finite element and boundary element method of Sinha (1997).

Measured values were available for the settlement of the foundation, the percentage of load carried by the piles, the maximum load carried by a pile in the group and the minium load carried by a pile in the group. The results of the six different analysis methods are shown in the bar chart of Figure 11 compared with the measured values and the values reported by Franke *et al*. (1994).

Figure 9 (right): DG Bank building, Westendstrasse 1, Frankfurt (courtesey of H.G. Poulos)

Figure 10 (below): Cross-section and pof DG Bank Building, Westendstrasse 1.





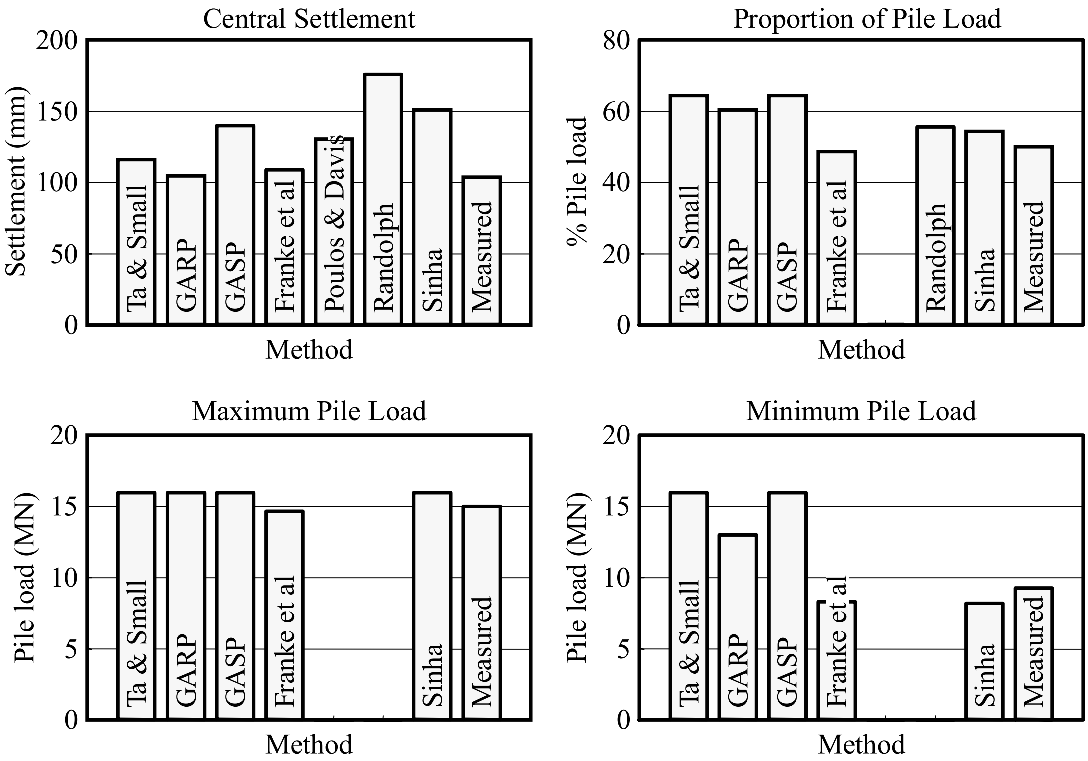


Figure 11: Comparison of measured and computed results (DG bank building, Westendstrasse 1)

From the figure it may be seen that:

- Most of the methods over-predicted the settlement of the foundation. However this depends on the soil modulus chosen, and it can only be concluded that most of the methods gave a reasonable estimate of the settlement for the adopted uniform soil stiffness of 62.4 MPa.
- Most of the methods over-predicted the percentage of load carried by the piles, although the calculated values are acceptable from a design point of view.
- All of the methods that are able to give a prediction of pile load, suggest that the most heavily loaded pile is almost at its ultimate capacity, and this is in agreement with the measured value.
- For the minimum pile load, there is a considerable variation in the calculated results, with three of the methods indicating a much larger value than was measured.

These results show that, when some of the piles are carrying loads close to their capacity, there can be significant variability in the computed results, especially for simple methods and methods based on the theory of elasticity.

5 CONCLUSIONS

In the previous sections, several aspects of soil-structure interaction have been examined, and from the data presented the following may be concluded:-

(1) The use of spring or 'Winkler' models can lead to erroneous results and should not be used. Continuum models (for the soil) are a more rational way to model the foundation soil, and a linear or non-linear continuum model is desirable.

(2) When analysing rafts, or piled rafts, inclusion of the stiffness of the superstructure will reduce the differential deflections in the raft. The relative stiffness of the superstructure will determine the effect, but for very flexible structures the raft alone can be analysed without great error.

(3) Simple models, like those based on the finite layer method, can yield results of acceptable accuracy without the need to use full three dimensional numerical methods.

(4) Reasonable predictions of raft behaviour can be made for full scale structures using simple techniques, provided the loads placed on the piles are not such that pile yield or yield of the soil occurs.

6 ACKNOWLEDGEMENTS

Much of the material presented in this paper has come from the work of colleagues and past research students at the University of Sydney for which I am very grateful. Some of the many names are mentioned in the text have been included in the references. The work done by Professors E.H. Davis, Harry Poulos, John Booker, and John Carter and Dr Peter Brown in the field of soil-structure interaction has laid the foundations for the work currently being undertaken at the University of Sydney.

7 REFERENCES

Brown, P.T. 1969a. Numerical analysis of uniformly loaded circular rafts on elastic layers of finite depth. *Géotechnique*, 19(2):301-306.

Brown, P.T. 1969b. Numerical analysis of uniformly loaded circular rafts on deep elastic foundations. *Géotechnique*, 19(3):399-404.

Brown, P.T. 1975a. Strip footing with concentrated loads on deep elastic foundations. *Geotechnical Engineering*, 6:1-13.

Brown, P.T. 1975b. The significance of structure-foundation interaction. *Proc. 2nd Australia-New Zealand Conf. on Geomechanics*, Brisbane, IEAust, l.1:79-82.

Brown, P.T 1977. Strip Footings. *Lecture 7, Geotechnical Analysis and Computer Applications*, Department of Civil Engineering, The University of Sydney. 1

Brown, P.T. and Weisner, T.J. 1975. The behaviour of unfromly loaded piled strip footings. *Soils and Foundations.* 15(4):13-21.

Brown, P.T. & Yu, S.K.R. 1986. Load sequence and structure-foundation interaction. *Journal of Structural Engineering, ASCE*, 112(1): 481-488.

Butterfield, R. and Banerjee, P.K. 1971. The elastic analysis of compressible piles and pile groups. *Géotechnique*, 21(1): 43-60.

Cheung, Y.K. and Nag, D.K. 1968. Plates and beams on elastic foundations - linear and non-linear behaviour. *Géotechnique*, 18:250-260.

Cheung, Y.K. And Tham, L.G. and Guo, D.J. 1988 Analysis of pile group by infinite element layer method. *Géotechnique*, 38(3):415-431.

Cheung,Y.K. and Zienkiewicz, O.C. 1965. Plates and tanks on elastic foundations - an application of finite element methods. *Int. Jl. Solids and Structs.* 1:451-461.

Chow, Y.K. 1986. Analysis of vertically loaded pile groups *Int. Jl. For Numerical and Analytical Methods in Geomechanics,* 10:59-72.

Clancy, P. & Randolph, M.F. 1993. An approximate analysis procedure for piled raft foundations. *Int Jl. for Numerical and Analytical Methods in Geomechanics,* 17(12):.849-869.

Desai, C.S., Johnson, L.D. & Hargett, C.M. 1974. Analysis of pile supported gravity lock. *Jl. Geotechnical Division, ASCE,* 100(GT9):1009-1029.

Franke, E. 1991. Measurements beneath piled rafts. Keynote lecture *ENPC Conf. Paris*. 1-28.

Franke, E., Lutz, B. & El- Mossallamy, Y. 1994. Measurements and numerical modelling of high-rise building foundations on Frankfurt clay. *Vert. and Horiz. deformation of foundations and embankments, ASCE Geot. Spec. Pub. No. 40.* 2:1325-1336.

Fraser, R.A. & Wardle, L.J. 1975. A rational analysis of shallow footings considering soil-structure interaction. *Australian Geomechanics Journal,* 20-25.

Fraser, R.A. & Wardle, L.J. 1976. Numerical analysis of rectangular rafts on layered foundations, *Géotechnique*, 26(4):613-630.

Gorbunov-Possadov, M.I. and Serebjanyi, R.V. 1961. Design of Structures on Elastic Foundations*, Proc. 5th Int. Conf. on Soil Mech. and Found. Eng., Paris, France, July, 1961*, 1:643-648.

Gusmau Filho, J.A. & Guimaraes, L.J.N. 1997. Limit stiffness in soil-structure interaction of buildings. *Proc. 14th Int. Conf. on soil mechs and Found Eng., Hamburg,* 2:807-808.

Hain, S.J. and Lee, I.K. 1974. Rational analysis of raft foundation. *Jl Geotech. Eng., ASCE,* 100(GT7):843-860.

Hain, S. & Lee, I.K. 1978. The analysis of flexible raft-pile systems. *Géotechnique,* 28(1):65-83.

Hansbo, S. and Källström, R. 1983. A Case Study of Two Alternative Foundation Principles. *Väg-och Vattenbyggaren,* 7-8:23-27.

Hemsley, J.A. (ed) 1998. *Design Applications of Raft Foundations.* Thomas Telford, London. (Hemsley, J.A., Chapter 18 – *Developments in raft analysis and design.* 486-605.)

Hemsley, J.A. 2000. *Elastic Analysis of Raft Foundations.* Thomas Telford, London.
Hooper, J.A. 1973. Observations on the Behaviour of a Piled-Raft Foundation on London Clay. *Proc. Instn. Civil Engineers,* 55(2) 855-877. (Discussion 1974 57(2);547-552).
Horikoshi, K. and Randolph, M.F. 1997. Optimum design of piled raft foundations. *Proceedings of the 14th Int. Con. on Soil Mechs. and Found Eng., Hamburg.* 2:1073-1076.
Katzenbach, R. and Reul, O. 1997. Design and performance of piled rafts. *Proceedings of the 14th Int. Conf. on Soil Mechs. and Found Eng., Hamburg,* 4:2253-2256.
Katzenbach, R., Arslan,U., Gutwald, J. and Holzh≅user, J. 1997. Soil-structure-interaction of the 300m high Commerzbank tower in Frankfurt am Main. Measurements and numerical studies. *Proceedings of the 14th Int. Conf. on Soil Mechs. and Found Eng., Hamburg,* 2:1081-1084.
Kuwabara, F. 1989. An elastic analysis for piled raft foundations in a homogeneous soil. *Soils and Foundations,* 29(1):82-92.
Lin, D.G., Bergado, D.T. and Balasubramanium, A.S. 1999. Soil-structure interaction of piled raft foundation in Bangkok subsoil. *Eleventh Asian Regional Conf. on Soil Mech. Found. Eng*., Hong et al. Eds., 183-187.
Lee, I.K. 1975. Structure-foundation-supporting soil interaction analysis. S. Valliappan, S. Hain and I.K. Lee (eds.) *Soil Mechanics, Recent Developments*, Univ. of NSW, 255-294.
Lee, I.K. & Brown, P.T. 1972. Structure-foundation interaction analysis. *Jl. Structs. Div. , ASCE,* 98 (ST11):2413-2430.
Lopes, F.R. and Gusmao, A.D. 1991. On the influence of soil-structure interaction in the distribution of foundation loads and settlements. *10th Europ. Conf. SMFE, Florence*, 2:475-478.
Milovic, S.D. 1998. A comparison between observed and calculated large settlements of raft foundations. *Canadian Geotechnical Journal*, 35(2):251-263.
Ottaviani, M.1975. Three-dimensional finite element analysis of vertically loaded pile groups, *Géotechnique,* 25(2):159-174.
Poulos, H.G. 1975. Settlement analysis of structural foundation systems. *Proc. 4th S.E. Asian Conf. Soil Eng. Kuala Lumpur.*
Poulos, H.G. 1991. Analysis of piled strip foundations. G. Beer, J.R. Booker and J.P. Carter (eds) *Proc. 7th Int Conf. on Comp. Methods and advances in Geomechanics, Cairns, 6-10 May.* 1:183-191. Rotterdam, Balkema.
Poulos, H.G. 1994. An approximate numerical analysis of pile-raft interaction. *Int. Jl. for Numerical and Analytical Methods in Geomechanics,* 18(2):73-92.
Poulos, H.G. & Davis, E.H. 1980. *Pile foundation analysis and design.* John Wiley and Sons, New York.
Rajapakse, R.K.N.D. 1988. Interaction between a circular elastic plate and a transversley isotropic elastic half-space. *Int. Jl. for Numerical and Analytical Methods in Geomechanics*, 12(4):419-436.
Randolph, M.F. 1983. Design considerations for offshore piles. *ASCE Spec. Conf. Geot. Practice in Offshore Eng*., Austin, 422-439.
Randolph, M.F. 1994. Design methods for pile groups and piled rafts. *Proceedings of the XIII International Conference on Soil Mechanics and Foundation Engineering*, *New Delhi, India, January 5-10*, 61-82.
Reul, O. 1998 Soil-structure interaction of a piled raft foundation*. 12th Europen Young Geotechnical Engineers Conference*, Tallinn, Estonia, 1-12.
Selvaduri, A.P.S 1980. Elastic contact between a flexible circular plate and a transversely isotropic elastic halfspace. *Int Jl. of Solids & Structs.* 16(2):167-176.
Sinha, J. 1997. Piled raft foundations subjected to swelling and shrinking soils. *PhD thesis,* University of Sydney, Australia.
Small, J.C. 2002. FEAR User's Manual, Centre for Geotechnical Research, The University of Sydney, NSW 2006.
Small, J.C. and Booker, J.R. 1986. Finite layer analysis of layered elastic materials using a flexibility approach, Part 2 - Circular and Rectangular Loadings, *International Journal for Numerical Methods in Engineering,* 23:959-978.
Smith, I.M. 1970. A finite element approach to elastic soil-structure interaction. *Canadian Geotechnical Journal*, 7(2):95-105.
Ta, L.D. and Small, J.C. 1996. Analysis of piled raft systems in layered soils. *Int. Jl. for Numerical and Analytical Methods in Geomechanics*, 20:57-72.
Tham, L.G. Man, K.F. and Cheung, Y.K. 1988. Analysis of footing resting on non-homogeneous soil by double spline element. *Computers and Geotechnics.* 5:249-268.
Timoshenko, S. and Woinowski-Krieger (1959) *Theory of Plates and Shells.* McGraw-Hill.
Viggiani, C. 1998. Pile groups and piled raft behaviour. *Deep Foundations on Bored and Augered Piles, BAP III*, van Impe and Haegman (eds), Balkema, Rotterdam, 77-90.
Yao, Z.E. & Zhang, J.R. 1985. An assessment of the effects of structure/raft/soil interaction. *Proceedings of the 5th International Conference on Numerical Methods in Geomechanics, 1-5 April , Nagoya, Japan.* 2:813-819.
Zhang, B.Q. and Small, J.C. 1992. The analysis of rectangular rafts of finite flexibility subjected to concentrated loads, *Proceedings of the Sixth Australia-New Zealand Conference on Geomechanics, Christchurch,* 205-210.

Zhang, B.Q. and Small, J.C. 1994. Finite layer analysis of soil-raft-structure interaction. *Proceedings of the XIII International Conference on Soil Mechanics and Foundation Engineering*, *New Delhi, India, January 5-10,* 2:587-590.

Zhang, H.H. and Small, J.C. 2000. Analysis of capped pile groups subjected to horizontal and vertical loads, *Computers and Geotechnics*, 26(1):1-21.

Zhemochkin, B.N. and Sinitsyn, A.P. 1962. *Practical methods of designing foundation beams and slabs resting on an elastic foundation*. 2nd Ed., State Publishing House for Literature on Structures, Architecture and Structural Materials, Moscow.

Zhuang, G.M., Lee, I.K. and Zhao, X.H. 1991. Interactive analysis of behaviour of raft-pile foundations, *Proceedings of the Int. Conf. on Geotech. Eng. for Coastal Development – Theory and Practice on Soft Ground* - *GEO-COAST '91. 3-6 Sept Yokohama*. 1:759-764.