THE MECHANICS OF DISCONTINUA: ENGINEERING IN DISCONTINUOUS ROCK MASSES

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SUMMARY

Rock masses are distinguished from most other engineering materials by their inherently discontinuous nature and by the range of scales on which discontinuities occur within them. The paper highlights a number of concerns that these factors pose in engineering practice. It reviews the basic mechanics of discontinua and the historical development of the characterisation, testing and analytical and numerical techniques available to the engineer working with discontinuous rock. The practical application of these techniques is illustrated by examples of their use in underground construction, caving methods of mining and hot dry rock geothermal energy exploitation. Despite the difficulties that still arise in engineering in discontinuous rock masses, it is shown that quite remarkable advances have been made in the last 40 years.

1 INTRODUCTION

Rock engineering is concerned with the investigation, design, construction and performance of engineered structures built on, in or of rock. Despite the long history of the use of rock as a construction material, the development of the science of rock mechanics and of a mechanics-based rock engineering methodology occurred only relatively recently. In Europe, and in Australia, the initial development of these approaches appears to have been associated largely with civil engineering projects and especially with hydro-electric power schemes (Jaeger, 1972). This does not mean that developments in rock mechanics were ignored by the mining industry. The problems of "rock pressure" and mining-induced subsidence, for example, had been the subject of systematic engineering study since the late 19th century (Hood and Brown, 1999).

Rock engineering is rendered more challenging than most other branches of engineering by the complexity and variability of the basic materials involved. In particular, rock masses are distinguished from other engineering materials by the presence of a range of inherent discontinuities (micro-fractures, joints, bedding planes, schistosity, faults) that may control their engineering behaviour. Whereas soils are discontinuous on the scale of a particle or grain and may contain bedding planes, rock masses may be discontinuous on a wider range of scales from the microscopic to the continental. Furthermore, discontinuities have fractal or self-similar geometries over several orders of magnitude (Hobbs, 1993).

Perhaps the first systematic study of rock masses as discontinuous engineering materials was that initiated by Josef Stini in Austria in the 1920s. An early manifestation of the emergence of rock engineering as a specialist engineering discipline was the publication in 1929 of the first volume of the journal *Geologie und Bauwesen* (Geology and Construction), edited by Stini. A feature of the work of what came to be known as the Austrian School of Rock Mechanics was the measurement, description and analysis of the discontinuous nature of rock masses (Müller, 1963). Annual colloquia on rock mechanics have been held in Austria since the 1950s and in the United States of America since 1956. The International Society for Rock Mechanics (ISRM) was formed under the leadership of the Austrian, Leopold Müller, in 1962. In Australia, the main impetus for the development of rock engineering methods and expertise was the design and construction of the monumental Snowy Mountains Hydro-electric Scheme in the period 1949-1969. It was during this period that rock mechanics research and teaching became established in Australia's universities, distinctive research programs were started in the CSIRO and rock mechanics programs were established in Australia's mines (Brown, 1991).

The purpose of this paper will be to explore the development of rock engineering techniques for dealing with this inherently discontinuous nature of rock masses in the approximately 40 year period since the early 1960s when modern rock mechanics and rock engineering may be regarded as having emerged as an identifiable engineering science and an identifiable engineering specialisation, respectively (Brown, 1993, 1999; Hood and Brown, 1999). This 40 year period coincides almost exactly with the author's professional life. Emphasis will be placed on contributions made to this development in Australia, although not to the exclusion of those made elsewhere. In particular, the seminal contributions made by the author's teacher and mentor, D. H. Trollope, and by J. C. Jaeger, will be highlighted. The advances of the last 40 years will be illustrated by examples of the application of modern rock engineering methods in underground civil engineering construction, underground mining and energy resource exploitation.

2 DISCONTINUITIES IN ROCK ENGINEERING

In rock mechanics and rock engineering it is common to distinguish between the apparently intact **rock material** and the discontinuous **rock mass**. The engineering response of the rock material itself may be complex and difficult to describe theoretically. On a microscopic scale, a given rock may consist of an aggregation of grains of minerals having quite different physical properties (e.g. granite). It may contain inter- and intra-granular microcracks and may have anisotropic and/or non-linear mechanical properties. In the usual engineering case of rock in compression, a particular complication arises from the friction mobilised between the surfaces of microcracks which serve as the sites of crack initiation and extension. This causes the rock material compressive strength to be highly sensitive to confining stress.

The general term **discontinuity** is used to describe any mechanical discontinuity in a rock mass having zero or low tensile strength. It is the collective term used for most types of joints, weak bedding planes, weak schistosity planes, fractures and faults. **Joints** are the most prevalent type of natural discontinuity. A joint is a break of geological origin in the continuity of a body of rock along which there has been no visible displacement. A group of sub-parallel joints is called a **set** and several joint sets may intersect to form a **joint system**. Joints may be open, filled or healed.

Discontinuities may influence the engineering responses of rock masses in a variety of ways including:

- providing planes of low shear strength on which slip might occur;
- reducing the overall shear and tensile strengths of the rock mass;
- reducing the overall stiffness of the rock mass;
- rendering the overall mechanical response of the rock mass discontinuous in the sense that individual blocks may be free to rotate or to translate with associated slip and/or separation at block interfaces;
- introducing a wide range of potential failure mechanisms such as unravelling, toppling, slip or the gravity fall of blocks or wedges not present in a continuum;
- influencing the stress distribution within the rock mass because of their low stiffnesses and strengths;
- attenuating, reflecting and refracting stress waves arising from blasting and other sources;
- controlling to a large extent the fragmentation achieved by excavation processes and
- because their permeabilities are orders of magnitude higher than those of intact rocks, providing major conduits for the flow of fluids through most rock masses.

Figure 1 shows a well-known simplified representation of the influence exerted on the selection of a rock mass behaviour model by the relation between discontinuity spacing and the size of the problem domain. The behaviour of the intact material may be of concern when considering the excavation of rock by drilling (Figure 2a) although the discontinuities present may control the block size distribution and fragmentation achieved, or when considering the stability of excavations in good quality, brittle rock subject to rock burst conditions. The behaviour of single discontinuities or of a small number of discontinuities, may be of paramount importance in considerations of the equilibrium of blocks of rock formed by the intersections of a number of discontinuities and a face of an excavation (Figure 2b), and in cases in which slip may occur on a penetrative fault.

As the ratio of the discontinuity spacing to the size of the problem domain increases further, it may become necessary to consider the rock mass as an assembly of discrete blocks or a discontinuum (Figure 2c). The essential distinction between continuum and discontinuum behaviour is that displacement fields need not be physically continuous in discontinua. Individual blocks may be free to rotate or to translate with associated slip and/or separation at block interfaces. Finally, it is sometimes necessary to consider the global response of a jointed rock mass in which the discontinuity spacing is small on the scale of the problem (Figure 2d) as in a high open pit slope. In these cases the rock mass may be treated as an equivalent or pseudo-continuum.

3 THE BASIC MECHANICS OF DISCONTINUA

As has been indicated, basic features of the mechanics of discontinua are that displacement fields are not continuous and that individual blocks may be free to rotate and translate. These factors introduce a number of potential modes of deformation and failure that are not usually experienced in continua. These features of the mechanics of discontinua will be illustrated by analytical approaches to apparently simple problems – Jaeger's single plane of weakness theory and its application to regularly jointed rock masses, Trollope's systematic arching theory and the analysis of the stability of roof beams in jointed rock.

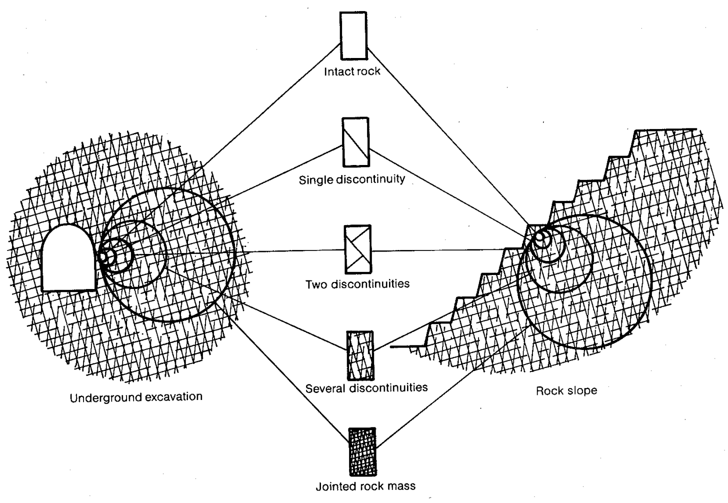


Figure 1: Simplified Representation of the Influence of Scale on the Rock Mass Behaviour Model used in Designing Underground Excavations or Rock Slopes (Hoek, 1983).

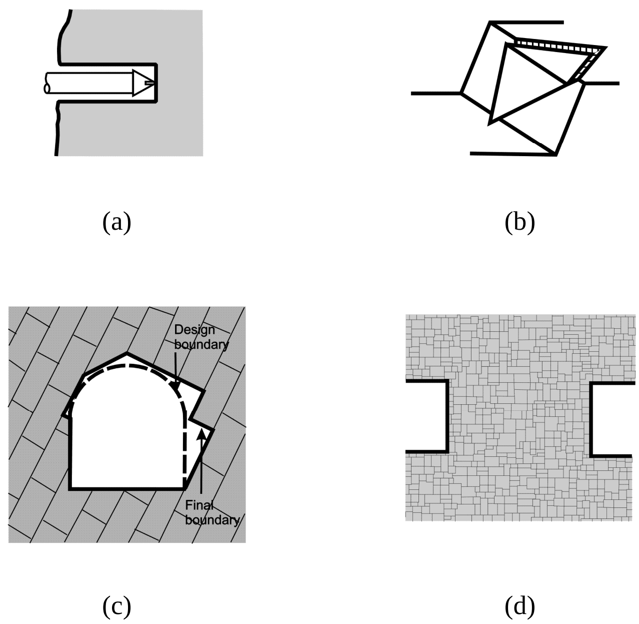


Figure 2: Effect of Scale on Rock Mass Response. (a) Rock Material Failure During Drilling; (b) Wedge Failure in a Rock Slope; (c) Discontinuities Controlling the Final Shape of an Excavation; (d) Mine Pillar Operating as a Pseudo-continuum (after Brady and Brown, 1993).

3.1 JAEGER'S SINGLE PLANE OF WEAKNESS THEORY

The starting point for analyses of the influence of discontinuities on rock mass behaviour is what has become known as Jaeger's single plane of weakness theory (Jaeger, 1960). Figure 3a shows a two-dimensional representation of a rock specimen containing a single plane of weakness AB whose normal is inclined at an angle β to the direction of the major principal stress. Jaeger (1960) actually considered the case in which the specimen contained a number of well-defined, parallel planes of weakness. Assume that the limiting shear strength of the plane of weakness can be described by a linear Coulombic shear strength law with a cohesion, c_w, and an angle of friction, ϕ_w. Slip on the plane of weakness will become incipient when the shear stress on the plane becomes equal to, or greater than, the shear strength. The normal and shear stresses on the plane AB are given by:

$$\sigma_n = \frac{1}{2}(\sigma_1 + \sigma_3)\cos 2\beta \tag{1}$$

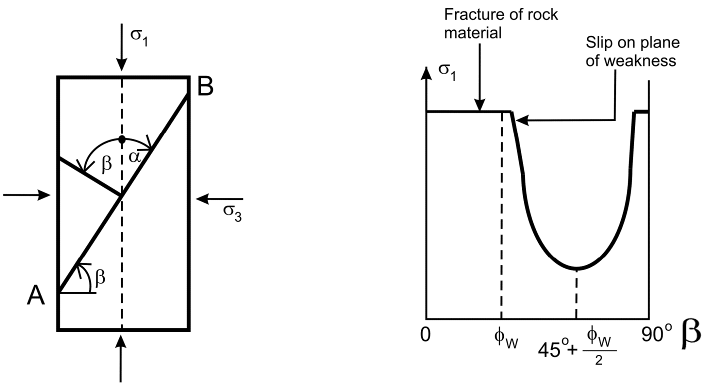
and

$$\tau = \frac{1}{2}(\sigma_1 - \sigma_3)\sin 2\beta \tag{2}$$

Substitution for σ_n in the shear strength equation and rearranging gives the criterion for slip on the plane of weakness as

$$(\sigma_1 - \sigma_3)_s = \frac{2(c_w + \sigma_3 \tan\varphi_w)}{(1 - \tan\varphi_w \cot\beta)\sin 2\beta} \tag{3}$$

The principal stress difference required to produce slip tends to infinity as $\beta \rightarrow 90^o$ and as $\beta \rightarrow \phi_w$. Between these values of β, slip on the plane of weakness is possible, and the stress at which slip occurs varies with β according to equation 3. The minimum strength occurs at $\beta = 45^o + \phi_w/2$. For values of β approaching 90^o and in the range 0^o to ϕ_w, slip on the plane of weakness cannot occur and so the peak strength of the specimen for a given value of σ_3, must be governed by some other mechanism, probably shear fracture through the rock material in a direction not controlled by the plane of weakness. The variation of peak strength with the angle β predicted by this theory is illustrated in Figure 3b.



(a) (b)

Figure 3: Jaeger Single Plane of Weakness Theory (after Jaeger, 1960).

Following analyses used by the Austrian School (John, 1962) and by Bray (1966, 1967), Hoek and Brown (1980) showed how the strength of rock masses containing multiple sets of intersecting joints could be estimated by applying this theory in several parts. Suppose that in two dimensions a rock mass can be represented as containing three sets of joints having the same shear strength characteristics, mutually inclined at 60^o as shown in Figure 4. Applying the Jaeger single plane of weakness theory to each set of joints and superimposing the results produces the peak shear strength characteristic shown in Figure 4. If one of the discontinuities has a lower shear strength than the others, the behaviour of the mass will become less closely isotropic. A modification of this approach has also been applied in cases in which shearing can take place on a plane passing partly through a discontinuity and partly through rock material (John, 1962; Brown, 1970).

Amadei (1988) pointed out that analyses of this type do not adequately describe the strength of regularly jointed rock masses because they are plane stress solutions which do not allow fully for the three-dimensional orientations of joints and of the applied stress field. Amadei (1988) developed a three-dimensional solution in which the three principal stresses could vary assuming tensile as well as the more usual compressive values. The joints were assumed to have zero tensile strength and to obey a Coulombic shear strength law. The strength of the rock material was assumed to described by the Hoek-Brown empirical peak strength criterion for intact rocks (Hoek and Brown, 1980). Amadei (1988) showed that the intermediate principal stress can have a considerable influence on the peak strength of a regularly jointed rock mass.

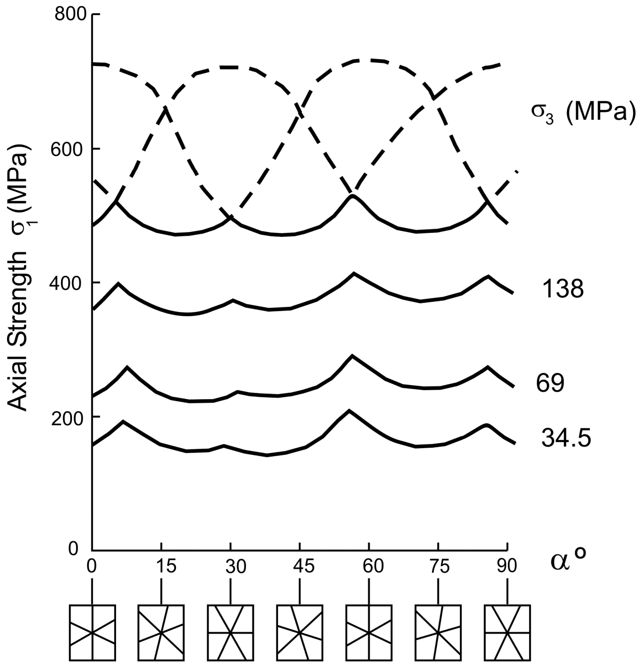


Figure 4: Jaeger Single Plane of Weakness Theory Applied to a Rock Mass Containing Three Sets of Similar Discontinuities Mutually Inclined at 60° (after Hoek and Brown, 1980).

3.2 TROLLOPE'S SYSTEMATIC ARCHING THEORY

Beginning with his theoretical and experimental studies of the distribution of self-weight stresses under a symmetrical wedge of sand, D. H. Trollope (1956, 1957, 1968) developed a theory of the mechanics of discontinua, or clastic mechanics, in which the particulate nature of soil and rock masses was modelled explicitly. In the first PhD thesis on a geomechanics topic submitted to an Australian University, Trollope (1956) developed his systematic arching theory to model the non-uniform distribution of stresses developed under a symmetrical wedge of sand as the base was allowed to deform. In two dimensions, the sand mass was represented as an assembly of rigid, mono-sized discs. The six contact forces between adjacent discs were taken to be directed along the lines joining the disc centres (Figure 5a). Each disc was of weight w, and the angle θ was called the distribution angle. As the base deforms, the process of arching causes reductions in some of the contact forces and the eventual opening of gaps between adjacent particles. Trollope introduced an **arching factor, k,** to describe the degree of arching which, for mathematical convenience, was assumed to be constant throughout the mass. For no arching, k = 1 and all the R and N forces in Figure 5c vanish. For full arching, k = 0 and the Q or P forces vanish (Figure 5d).

The initial systematic arching theory for sand was extended subsequently to cases in which the distribution angles were other than the 30° applying for discs, and to the analysis of self-weight stresses in soil and rock slopes (Trollope, 1957, 1962), rock foundations (Trollope and Brown, 1968) and underground excavations in jointed rock (Trollope, 1966). In the later analyses, the influence of friction at particle contacts was allowed for. Further development and application of the simple theory was limited by the need to assume, for a given problem, generally uniform particle shapes and sizes, small displacements (although even very small displacements produced dramatic effects), a constant distribution angle and a constant arching factor. To obtain solutions to engineering problems in which some of these factors are varied generally requires the use of computer-based numerical models. However, Trollope's mathematically simple clastic mechanics approach, as well as being of major conceptual importance, was able to provide new insights into a number of practical problems, including the phenomenon of arching in discontinua. Subsequently, Savely (1987) showed how the distribution angle and the arching factor can be treated as variables in calculating self-weight stress distributions for use in probabilistic slope stability analyses.

Trollope's analyses were carried out in terms of inter-particle contact forces. The continuum mechanics concept of stress at a point could be applied to his results only when the particle size was assumed to be vanishingly small on the scale of the problem. This led Trollope to question the physical meaning of the continuum or differential concept of stress in discontinua and to propose an integral definition in its place (Trollope, 1968; Trollope and Burman, 1980). Trollope was one of the first individuals to insist on the need to consider the particulate nature of geomaterials and to develop a systematic theory of discontinua. His pioneering contributions have probably not been accorded the international recognition they merit, although it was pleasing to note that Fairhurst (2003) gave him due credit in his Leopold Müller Award lecture given to the recent 10th Congress of the International Society of Rock Mechanics.

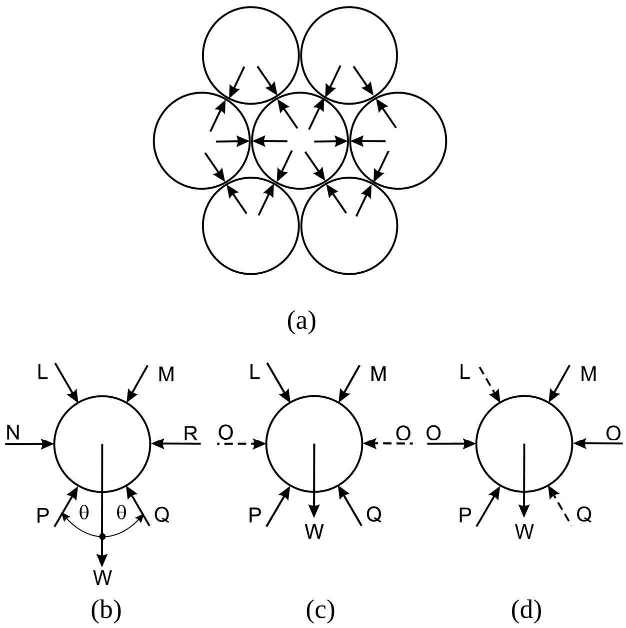


Figure 5: Trollope's Original Systematic Arching Theory

3.3 ROOF BEAMS IN JOINTED ROCK

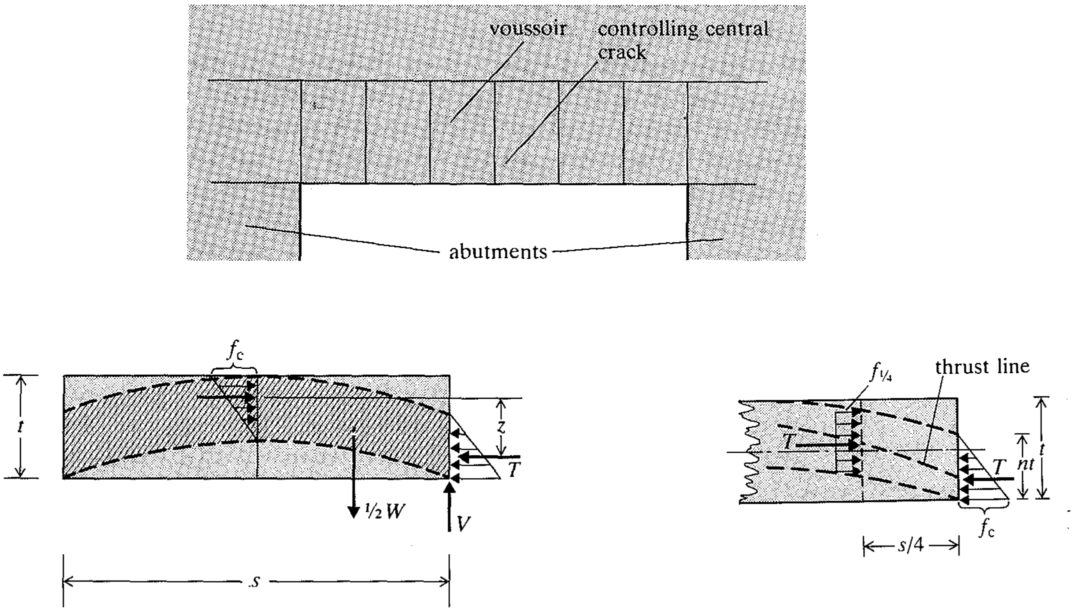
The apparently simple problem of the stability of a roof beam in jointed rock has been studied for well over 100 years (Fayol, 1885; Bucky, 1934; Evans, 1941; Trollope, 1966; Beer and Meek, 1982; Pells et al., 1991; Brady and Brown, 1993; Sofianos, 1996; Diederichs and Kaiser, 1999; Pells, 2002). The basic plane strain problem to be considered here is illustrated in Figure 6a. It has been found that the central crack or joint controls the deformation of the discontinuous beam so that the problem may be analysed in terms of the geometry and forces shown in Figure 6b. An examination of the problem geometry suggests that, in addition to the inability to transmit tension between the blocks in the beam, three failure modes are possible (Brady and Brown, 1993):

- shear at the abutment when the limiting shear resistance, T tan ϕ, is less than the required abutment vertical reaction, $V = W/2$;
- crushing at the hinges formed in the beam crown and lower abutment contacts and
- buckling of the roof beam with increasing eccentricity of the lateral thrust and a consequent tendency to form a "snap-through" mechanism.

The problem is statically indeterminate. In order to develop a solution, a number of assumptions have to be made about load distributions and lines of action of resultant forces. In particular, the end load distributions are assumed to be triangular and to operate over a portion, n t, of the beam depth, t, and an assumption has to be made about the locus of the horizontal reaction force or line of thrust which is shown as a parabolic arch in Figures 6b and c. Moment equilibrium of the free body diagram shown in Figure 6c produces an expression for the maximum compressive stress acting in the beam, f_c, in which n and the moment arm, z, are unknowns. Solving for the three modes of instability involves determination of f_c, n and z. In some applications, diagonal shear failure through the rock is a fourth possible failure mechanism.

With deformation of the beam and the development of the horizontal reaction force, there will be elastic shortening of the beam and thrust arch, and a change in the value of the moment arm, z. To calculate the elastic shortening and central deflection of the arch, a value of the *in situ* deformation modulus of the beam is required. It is also necessary to make an assumption about the distribution of axial compressive stress over the longitudinal section of the beam. Until recently, most analyses of the problem assumed a bi-linear variation but numerical analyses carried out by Diederichs and Kaiser (1999) showed that a quadratic variation may provide a better approximation for the simple, two member voussoir beam. Unless a value of n or z is assumed (as Evans (1941) did in putting n = 0.5 in his solution), some form of incremental solution is required to give equilibrium values of n and z corresponding to a minimum value of f_c.

Having obtained equilibrium values of n and z, the factors of safety against the three modes of failure can then be calculated. It is found that the value of n drops below 0.5 as critical or unstable beam geometry is approached.



(a)

(b)

(c)

Figure 6: Voussoir Beam Geometry and Load Specification for Roof Beam Analysis (Brady and Brown, 1993).

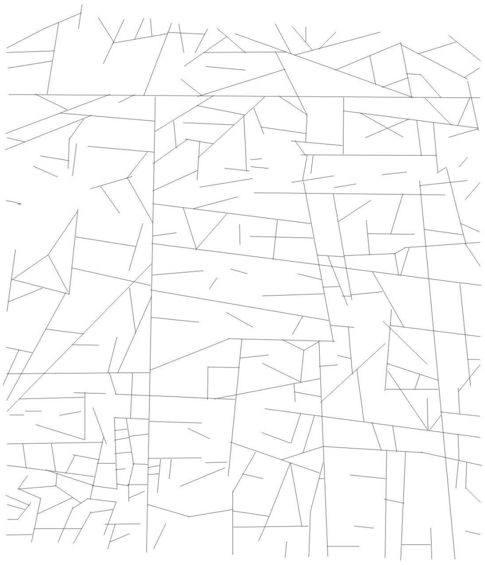
Most solutions (e.g. Brady and Brown, 1993; Diederichs and Kaiser, 1999) have assumed that the mechanics of the beam are controlled by the central crack and that the vertical deformations are small. In a more exhaustive analysis, Sofianos (1996) allowed for the possibility of large vertical deformations and used a numerical method to evaluate n. Sofianos (1996) obtained values of n that are considerably lower than those determined in other solutions. Sofianos (1996) used a parametric analysis to produce a single non-dimensional graph in which the loading and mechanical properties of the beam are related to its geometry for a range of factors of safety for the three modes of failure.

4 ROCK MASS CHARACTERISATION

Before any analysis of rock mass response can be carried out for a given rock engineering project, it is necessary that the discontinuities in the rock mass be characterised and their properties established. The ISRM Suggested Methods for the Quantitative Description of Discontinuities in Rock Masses identifies ten parameters as being of primary importance – orientation, spacing, persistence, roughness, wall strength, aperture, filling, seepage, number of sets and block size. The author has reviewed elsewhere the developments that have taken place in the collection, analysis and use of these data (Brown, 2003) and so the exercise will not be repeated here. Nevertheless, a small number of points should be made in the current context.

First, it is important to note that significant contributions were made to site investigation and rock mass characterisation techniques and methodology by the team of engineering geologists working on the Snowy Mountains Hydro-Electric Scheme (e.g. Moye 1955, 1959, 1967). Interestingly, that team included the recipient of the John Jaeger Memorial Award for 1996, D. H. Stapledon (Stapledon, 1961). Second, as illustrated in Figure 7, the collection, processing, storage and presentation of discontinuity data have benefited greatly from the advances that have been made in digital technologies in recent years (Brown, 2003; Harries, 2001). Nevertheless, the data collection task is still one that depends greatly on the training, knowledge and skill of the engineering geologist (Stapledon, 1996). Third, the discontinuous nature of rock masses is now represented through models which simulate the rock mass structure using measured values and statistical distributions of a number of the parameters listed above (e.g. Dershowitz, 1995). Such realisations of rock mass structure are used as the basis of numerical stability analyses, fluid flow analyses and fragmentation studies. A recent development in this area has been the introduction of a hierarchical modelling approach in which the order in which discontinuity sets were formed and the associated discontinuity termination data are reproduced (Harries, 2001; Harries and Brown, 2001).

(a)



(b)

Figure 7: Discontinuity Network Kangaroo Point, Brisbane: (a) Photograph; and (b) Digitised Discontinuity Trace Map (Harries, 2001).

5 MECHANICAL PROPERTIES OF DISCONTINUOUS ROCK MASSES

Because of the nature, scale, anisotropy and variability of discontinuous rock masses, their basic mechanical (load-displacement or stress-strain) properties are difficult to measure directly. Such measurements have been made using *in situ* loading tests and large-scale laboratory tests. More often, indirect methods are now used to supplement or replace direct methods. They include geophysical methods, model tests, combining intact rock and discontinuity properties using analytical or computational methods, and back analysis using field observations.

Large-scale *in situ* multiaxial loading tests provide an obvious direct approach. However, they suffer from some equally obvious disadvantages. One is their high cost, although this may not be as great a disadvantage as is often supposed, particularly when the costs of testing are considered in relation to those of the project as a whole. A more subtle difficulty concerns the reproducibility and comparability of results. It is often difficult to prepare *in situ* test specimens that are sufficiently similar to permit the results to be compared. If, as is often the case, the rock mass is anisotropic or heterogeneous, large numbers of tests may be required, accentuating the problem of costs. A classic example of multiaxial *in situ* testing is provided by the tests carried out at the Kurobe IV arch dam, Japan, in late 1961 and early 1962 (John, 1962; Müller-Salzburg and Ge, 1983).

Large-scale laboratory tests provide an alternative to *in situ* tests. Generally, it is considered that these tests can be used to determine overall rock mass properties only if the sample contains five to ten joints from each joint set present (ISRM Commission, 1989). The major problems encountered are in taking the samples and in handling, transporting and preparing them for testing so that they remain essentially undisturbed. A classic study of this type was that carried out by Jaeger (1970) on 152 mm diameter samples of the Panguna andesite from the Bougainville Copper open pit mine site, Papua New Guinea. This set of data was used in the formulation of the original version of the Hoek-Brown empirical strength criterion (Hoek and Brown, 1980). The best example of a large-scale laboratory test facility known to the author is that at the University of Karlsruhe, Germany (Natau et al., 1983, 1995; Mutschler and Natau, 1991). Specimens with dimensions of up to 0.62 m x 0.62 m x 1.20 m may be tested in standard ($\sigma_2 = \sigma_3$) or true ($\sigma_2 \neq \sigma_3$) triaxial compression under variable stress paths.

Model tests have been used to investigate deformation and failure mechanisms in discontinua (e.g. Brown, 1970; Brown and Trollope, 1970; John, 1970; Rosengren and Jaeger, 1968), develop general theories of rock mass behaviour (e.g. Brown, 1970; Ladanyi and Archambault, 1972) and, where the laws of similitude are applied correctly, to model a particular rock mass or prototype behaviour (e.g. Fumagalli, 1968; Liu et al 2003; Natau et al, 1983). The following failure mechanisms have been observed in laboratory biaxial and triaxial compression tests on models of jointed rock: shear fracture of initially intact rock material, cleavage or induced tensile fracture of initially intact material, slip on a single discontinuity, slip on multiple discontinuities, composite shear failure partly through intact material and partly along discontinuities, rotation of individual blocks or groups of blocks, and the development of wide shear zones, sometimes with kink bands (Brown, 1970; Lananyi and Archambault, 1972). Peak strength envelopes obtained in *in situ*, large-scale laboratory and model tests on discontinuous rock masses are usually non-linear.

Finally, analytical methods such as those outlined above (Amadei, 1988; Bray, 1967; John, 1962) and numerical models may be used to make assessments of the overall mechanical properties of a regularly jointed rock mass. Analytical formulations are usually concerned only with peak, or perhaps residual, strength conditions and cannot model the complete stress-strain behaviour of the rock mass. Well-formulated and applied numerical models are able to take account of a wider range of mechanisms of deformation and to model stress-strain behaviour more completely. The distinct or discrete element models to be discussed below are particularly powerful in this regard (e.g. Cundall, 1987, 2001).

Because of the difficulty of measuring or modelling accurately the strength and deformability of discontinuous rock masses, empirical methods have been used to relate strength and deformability to some measure of rock mass quality. Perhaps the best known of these methods is the empirical strength criterion developed by Hoek and Brown (1980, 1997). The history of the development of the Hoek-Brown criterion and its latest version have been given by Hoek and Brown (1997) and Hoek et al (2002), respectively, and will not be repeated here. However, one important point will be made. The criterion was developed originally for use only in preliminary or sensitivity analyses, sometimes before a complete site investigation and the relevant testing had been carried out (Hoek and Brown, 1980). Since that time, the criterion has been applied to conditions which were not visualised when it was originally developed (Hoek and Brown, 1997). Some significant changes have been made to permit the application of the criterion to poorer quality rock masses than those for which it was originally intended (Hoek et al, 2002) and to coal (Medhurst and Brown, 1998). Nevertheless, the essential point remains that there are a number of problem types for which the criterion is unsuitable. It should be used only for "those rock masses in which there are a sufficient number of closely spaced discontinuities that isotropic behaviour involving failure on discontinuities can be assumed" (Hoek and Brown, 1997). It should not be used when the block size is of the same order as that of the structure being analysed or where failure is controlled by a single discontinuity or a small number of discontinuities.

6 NUMERICAL MODELLING

The advances made in computer science and technology in the 40 years under review have been spectacular. This, in turn, has provided the impetus for the development of a range of numerical methods that are able to provide solutions to otherwise intractable problems. Two- and three-dimensional numerical analyses of quite large and complex rock engineering problems are now carried out routinely in engineering practice. This brief overview of the available numerical methods draws on that of the author's former research student, Lanru Jing (2003).

Jing (2003) classifies the numerical methods that are most commonly applied to rock mechanics problems in the following manner (with some key references to each method having been added by the author):

Continuum methods

- the Finite Difference Method (FDM) (Detourney and Hart, 1999);
- the Finite Element Method (FEM) (Goodman et al., 1968; Zienkiewicz, 1977; Wittke, 1990) and
- the Boundary Element Method (BEM) (Watson, 1979; Crouch and Starfield, 1983; Beer and Watson, 1992).

Discontinuum methods

- Discrete Element Methods (DEM) (Cundall, 1971, 1987, 2001; Hart, 1993) and
- Discrete Fracture Network (DFN) methods (Dershowitz, 1995; Harries and Brown, 2001).

Hybrid continuum/discontinuum models

- Hybrid FEM/BEM (Beer and Watson, 1992; Elsworth, 1986);
- Hybrid DEM/BEM (Lorig et al, 1986);
- Hybrid FEM/DEM (Pan and Reed, 1991) and
- other hybrid models.

To these most commonly used methods must be added the more specialist methods such as key block theory (Goodman and Shi, 1985), Discontinuous Deformation Analysis or the implicit DEM (Shi and Goodman, 1985; Jing, 1998) and Cosserat theory (Mühlhaus, 1993) which is a continuum theory that allows points within the material to have three degrees of rotational freedom in addition to three degrees of translational freedom. In the continuum-based models listed, a discontinuous rock mass may be modelled as an equivalent continuum or, alternatively, the intact blocks and the discontinuities, or more correctly some of the discontinuities, may be modelled explicitly. Figure 8 shows how a discontinuous rock mass may be represented using FDM or FEM, BEM and DEM methods.

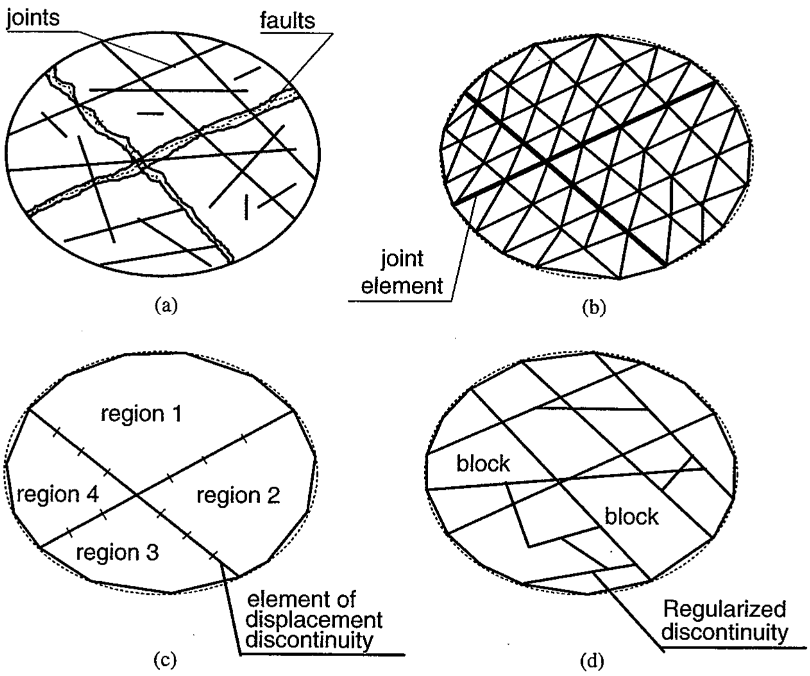


Figure 8: (a) Discontinuous Rock Mass Represented by (b) FDM or FEM, (c) BEM, and (c) DEM (Jing, 2003).

The most valuable and widely used method of modelling the engineering responses of discontinuous rock masses are **discrete element methods**. These methods are distinguished from other methods that have interfaces between elements on which slip may occur by allowing finite displacements and rotations of discrete bodies, including complete detachment, and recognising new contacts automatically as the calculation progresses. The term **distinct element method** was introduced by Cundall and Strack (1979) to refer to a particular discrete element scheme that uses deformable contacts between elements and explicit time-marching to solve the equations of motion directly. The elements themselves may be rigid or deformable. There are a number of distinct element codes available but the most widely used are the two-dimensional code, UDEC, and the three-dimensional code, 3DEC, developed from Cundall's original formulations (Hart, 1993). Figure 9 shows the application of UDEC to the analysis of slope stability in a bedded and jointed rock mass.

The distinct element method was developed originally for a two-dimensional representation of jointed rock masses but has since been extended to studies of particle flow, microscopic mechanisms in granular materials, crack development in rock and concrete, and blasting mechanics. Some of these applications make use of the code PFC3D (Particle Flow Code in 3 Dimensions). The PFC3D model consists of a three-dimensional collection of rigid particles or balls. All particles are spherical but individual spheres may be clumped together to form particles of arbitrary shape. Newton's laws of motion provide the fundamental relations between particle motion and the forces causing that motion. The force system may be in static equilibrium - in which case there is no motion – or it may be such that it causes the particles to flow. The model also includes "walls" that provide boundaries to the simulations. The spheres and walls interact through the forces arising at contacts, assuming linear springs in the normal and shear directions. Sliding is allowed at the contacts. A detailed description of the distinct element method is given by Cundall and Strack (1979). An example of its application to particle flow in the block caving method of mining will be given below.

While numerical modelling is now able to provide solutions and insights not once obtainable, it does not always provide direct answers to rock engineering problems. In order to obtain useful and believable solutions it is necessary that problems be properly formulated and that the boundary conditions and material properties be well-defined. An important requirement is to verify that the numerical method being used can reproduce the mechanisms actually occurring in practice. In discussing the application of the distinct element method to rock engineering problems, Hart (1993) suggested that "the temptation to build a complex joint structure into the model should be resisted. A more practical approach is to perform simpler analyses which focus on identifying important mechanisms (e.g. modes of deformation or likely modes of failure). These analyses can form the basis to provide bounds for rock engineering design."

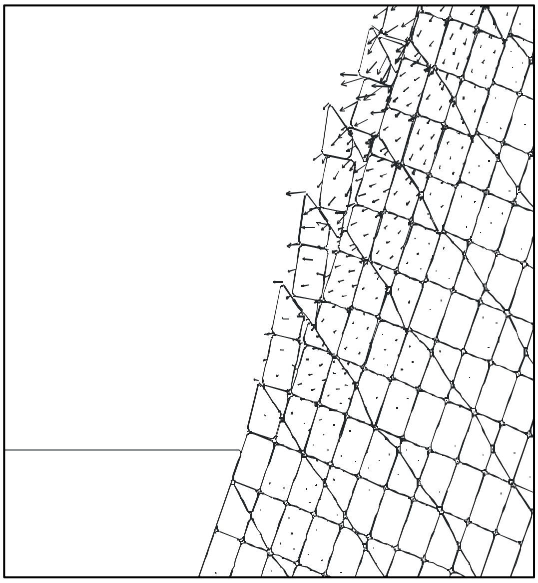


Figure 9: UDEC Simulation of Block Movement in an Excavated Slope.

7 UNDERGROUND EXCAVATION DESIGN

The methods of engineering in discontinuous rock outlined above are now used routinely in the design of large underground excavations including underground power stations, storage caverns, underground military installations, water and transportation tunnels, and other types of civil infrastructure. One of the most spectacular examples is the 62 m span Olympic Ice Hockey cavern constructed at Gjøvik, Norway, for the 1994 Winter Olympic Games (Barton et al, 1994). The cavern is in a jointed gneiss having an average RQD of 67% and Q values in the range 1–30 with a weighted average of 9. The rock cover of only 25-50 m posed particular design problems. Site investigations included two types of stress measurement, geotechnical core logging, cross-hole seismic tomography and Q system classification. Numerical modelling was carried out using UDEC. The predicted maximum roof deformation of only 4-8 mm reflected the effects of the high horizontal *in situ* stresses. Support and reinforcement consisted of 100 mm of wet process steel fibre reinforced shotcrete and systematic rock and cable bolting on alternating 2.5 and 5.0 m centre to centre spacings. The rock and cable bolts were untensioned and fully grouted. The maximum roof deformations measured when the 62 m span cavern was fully excavated were 7-8 mm (Barton et al., 1994).

Equally impressive have been the achievements of Dr P. J. N. Pells and his co-workers in the engineering design of a series of major excavations in the Hawkesbury sandstone, Sydney. Particularly notable are the donut shaped Sydney Opera House Car Park cavern and the double decker Eastern Distributor tunnel (Bertuzzi and Pells, 2002; Pells, 2002; Pells et al., 1991). Following geological mapping, geotechnical classification, *in situ* stress measurement and geotechnical model formulation, the analyses and designs used specially developed linear arch analysis, bedding plane shear stress and displacement calculations, and rock bolt and shotcrete capacity and design calculations.

The design and stability of the 17.5 m span Opera House Car Park cavern with only 6 m of rock cover posed a particular challenge. Crown sag was monitored carefully as the span was excavated progressively by stripping from an initial 6 m span heading. Figure 10 shows how the measured crown centre-line sag increased as the span increased compared with the design prediction (Pells, 2002). The typical geotechnical model and rock bolt pattern for the 3 km long double deck Eastern Distributor tunnel are shown in Figure 11. The span of the Eastern Distributor reached 24 m in one section.

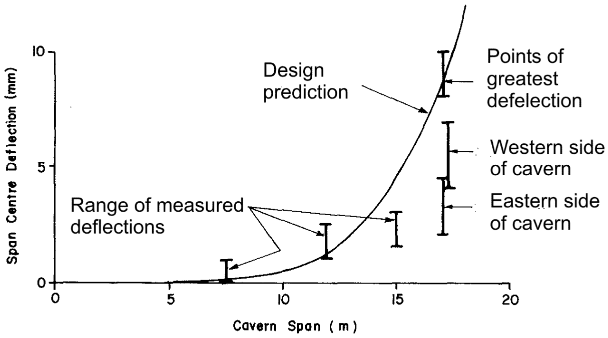


Figure 10: Measured and Predicted Roof Sag, Sydney Opera House Car Park Cavern (Pells, 2002).

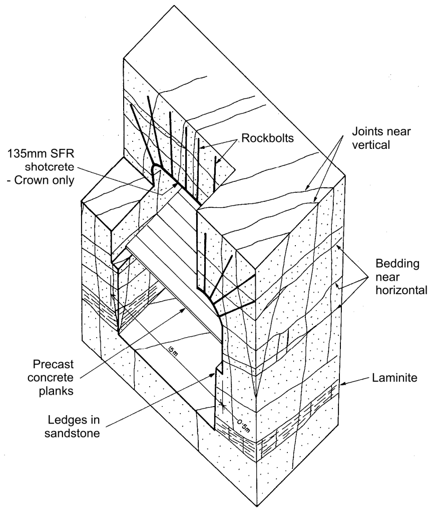


Figure 11: Eastern Distributor Double Deck Tunnel, Sydney (Pells, 2002).

8 BLOCK CAVING GEOMECHANICS

In the block caving method of underground mining and its variants such as panel caving (Figure 12), the orebody, or a block or a panel of ore, is undercut fully to initiate caving. The undercut zone is drilled and blasted progressively and some broken ore is drawn off to create a void into which initial caving of the overlying ore can take place. As more ore is drawn progressively following cave initiation, the cave propagates upwards through the orebody, block or panel until the overlying rock also caves and surface subsidence occurs. Figure 12 is a schematic illustration of modern mechanised panel caving at the El Teniente mine, Chile.

Block and panel caving may be used in massive orebodies which have large, regular “footprints” and either dip steeply or are of large vertical extent. It is a low cost mass mining method which is capable of automation. However, it is capital intensive requiring considerable investment in infrastructure and development before production can commence. Historically, block caving was used for massive, low strength and usually low grade orebodies which produced fine fragmentation. Where mining is mechanised, the low strength of the rock mass can place limitations on the practicable sizes of extraction level excavations. Furthermore, finely fragmented ore can “chimney” when drawn requiring the drawbells to be closely spaced so that undrawn “pillars” of broken ore do not form. These factors place limitations on the sizes of equipment that can be used. There is now an increasing tendency for the method to be used in stronger, deeper orebodies which produce coarser fragmentation than did the traditional applications of the method (Brown, 2003).

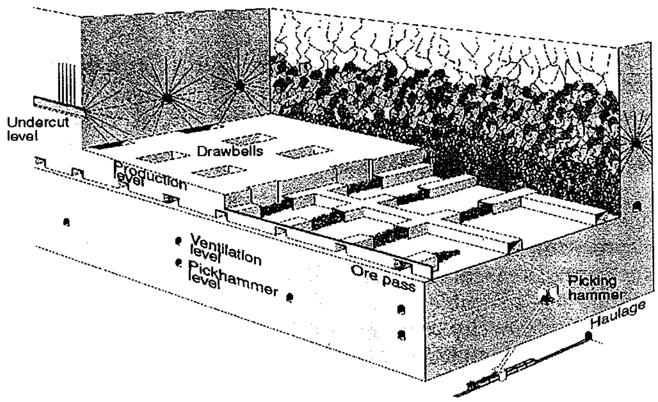


Figure 12: Mechanised Panel Caving, El Teniente Mine, Chile (Hamrin, 2001).

In addition to the usual financial and environmental risks associated with mining, cave mining projects involve a number of specific types of risk, many of them associated with the inherent and induced discontinuous behaviour of the rock mass. They include:

- **cavability** involving a prediction of the hydraulic radius (area/perimeter) of the undercut at which caving will initiate for a rock mass having given or estimated geotechnical properties;
- **cave propagation** or the ability of the cave to continue to propagate and not stall once caving has been initiated;
- **the degree of fragmentation** of the ore occurring as a result of the caving process and the ability to predict it for design purposes;
- **stability** over the design life and the need for support and reinforcement of mine excavations including undercut drifts, extraction level excavations, drawbells and items of mine infrastructure;
- **subsidence** to surface and its prediction and
- **major** operational hazards such as excavation collapses, mud rushes, rock bursts, air blasts and water and slurry inflows.

Risk assessment and management techniques may be used to deal with many of these issues in practice. In recent years, the author has been associated with studies of these issues as part of the industrially sponsored International Caving Study carried out through the University of Queensland's Julius Kruttschnitt Mineral Research Centre (Brown, 2003). Space limitations permit only a sample of the complex geomechanics issues involved to be discussed here. The first is the fundamentally important one of cavability. In the original block caving applications, caving of the ore was controlled by the pre-existing discontinuities in the rock mass. The structure found to be most favourable for caving was one in which a low-dip discontinuity set was augmented by two steeply dipping sets which provide conditions suitable for the vertical displacement of pre-formed rock blocks. In the deeper, stronger rock orebodies subject to higher *in situ* stress fields now being mined by the method, the dominant mechanisms are brittle fracture of the intact rock and slip on discontinuities, especially those that are flat dipping. This form of caving is sometimes referred to as **stress caving**.

As part of the International Caving Study Stage I reported by Brown (2003), Lorig and Cundall (2000) carried out a series of PFC3D simulations of particle flow towards drawpoints. The effects of interaction between drawpoints were studied using models having four adjacent drawpoints. The results presented here are for constant and equal draw from all drawpoints. In one set of runs, the spacing of the drawpoint centres was 13 m, the tops of the cones had radii of 5.73 m and they were separated from their closest neighbours by 1.54 m (1.54 + 2 * 5.73 = 13 m pitch). The distance across the diagonal between cone centre lines was 18.38 m. Therefore, the distance between cone rims across the diagonal was 6.92 m. In dimensionless form, the ratio of separation (between rims) to cone radius was either 0.27 or 1.21, depending on the path. Figure 13 shows that there is complete interaction between the drawpoints, with plug flow occurring essentially over the full ore column height (i.e. there is no dead zone).

In a second model, the drawpoints were spaced 18 m apart (Figure 14). The separation ratios here (using similar logic to the above) were 1.14 and 2.44. In this case, there was interaction only at some height above the draw level (depending on how well developed was the interactive draw zone or IDZ). At a late stage in the test, plug flow was seen at a height of about 1.5 times the cone radius above the draw level. Below this, there was a dead zone. Lorig and Cundall (2000) found that the flow pattern may be compared with that of a single drawpoint run with similar conditions.

The IDZ extended laterally only to about 50% of the cone radius. Thus, there appears to be some interaction between IDZs in the model shown in Figure 14, which, taken separately, would not intersect. In the model shown in Figure 13, adjacent IDZs would be expected to touch (or nearly touch) soon after drawing begins, so the observed strong interaction is not surprising.

Another PFC3D simulation demonstrated how very high contact forces can result when hangups occur through arching above a drawpoint. In the example shown in Figure 15, the black lines between particles indicate the directions of the contact forces and the line thicknesses indicate force magnitudes. In this case, the maximum force magnitude is 607 kN.

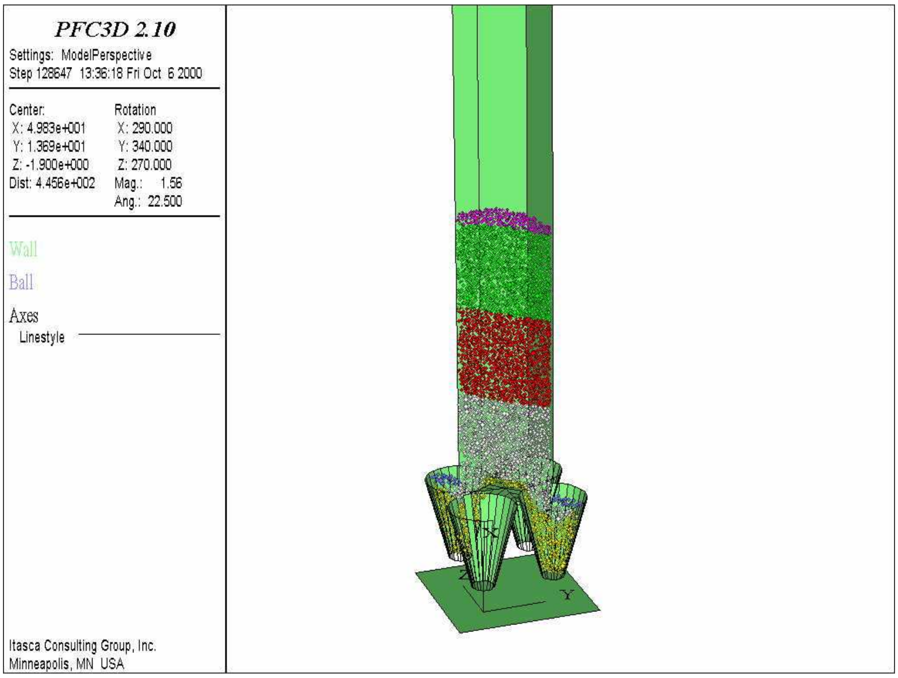


Figure 13: Four Drawpoint Model with 13 m Separation Between Drawpoints Showing Complete Interaction (Plug Flow) (Lorig and Cundall, 2000).

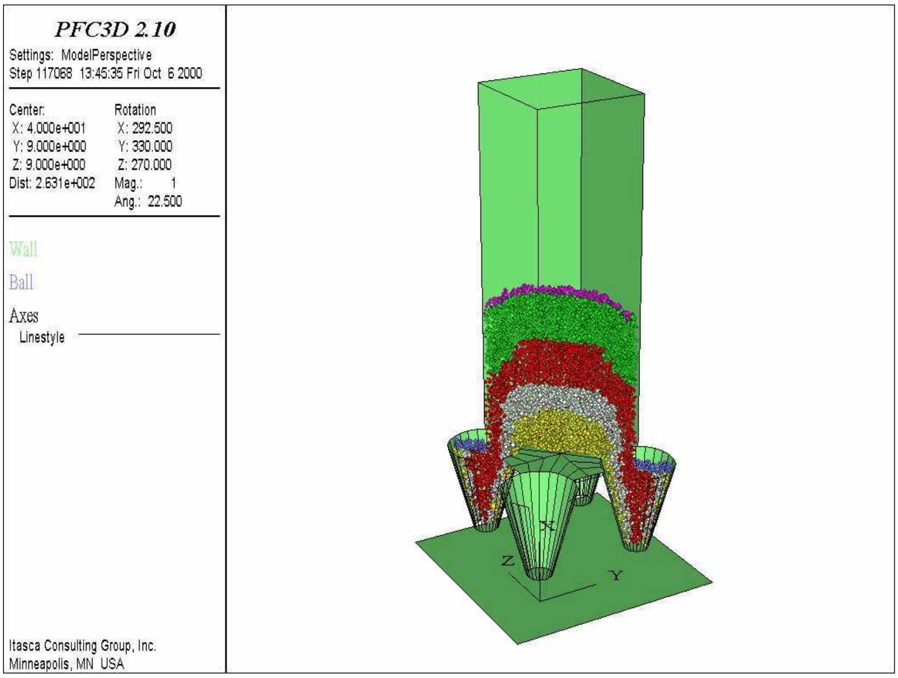


Figure 14: Four Drawpoint Model with 18 m Separation Between Drawpoints Showing Poor Interaction (Lorig and Cundall, 2000).

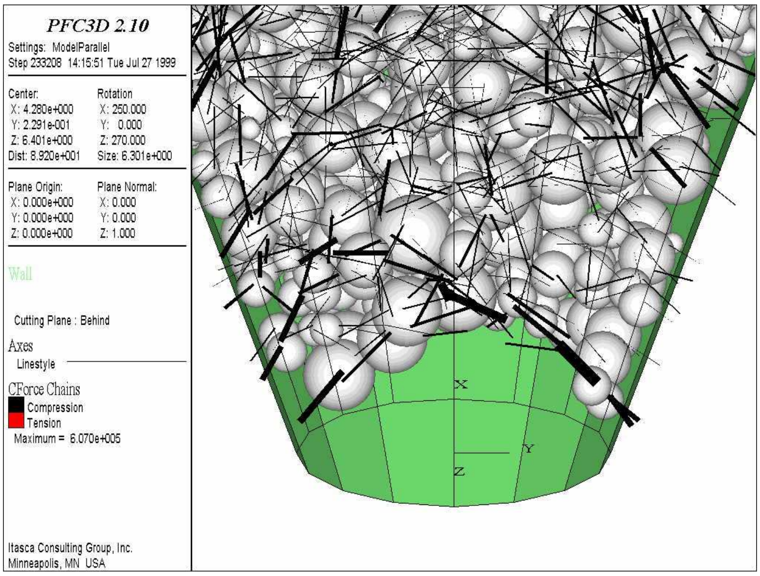


Figure 15: Draw Cone in PFC3D Model Shortly After Hangup Occurs (Lorig and Cundall, 2000).

9 HOT DRY ROCK GEOTHERMAL ENERGY

An attractive and plentiful source of renewable energy is the heat contained in high heat producing granites located 3 km or more below the Earth's surface. Since the 1970s, a number of research and demonstration projects on hot dry rock (HDR) geothermal energy have been carried out in France, Germany, Japan, Switzerland, the UK, the USA and, most recently, Australia. The HDR concept is illustrated in Figure 16. Two or more holes are drilled into the target granite using standard oil well drilling technologies. An underground heat exchanger is then developed by hydraulic stimulation techniques in which water and other special fluids are pumped down one hole under pressure. The water enters the pre-existing distribution of joints inducing shearing, and sometimes jacking, of joints which may be kept open naturally or by introducing artificial proppants. These opened joints form a network of fluid pathways in the hot granite. The size and orientation of the stimulated zone depends on the interaction of the joint system and the *in situ* stress field. Initially cold water may be circulated through the underground heat exchanger by injecting it through an injection well and recovering it through one or, preferably, more production wells. The superheated water brought to the surface under pressure is used to boil an organic liquid to produce vapour which drives a turbine to produce electricity (Figure 16). In a commercial HDR development, a system of multiple interacting injection and production wells would be required over an area of a few square kilometres.

The important influence of the *in situ* stress field on the engineering response of discontinuous rock masses was well illustrated in the hot dry rock geothermal energy project carried out by the Camborne School of Mines (CSM) in Cornwall, UK in the late 1970s and 1980s (Batchelor, 1984). During the course of injecting more than 300,000m^3 of water into 2 km deep boreholes in the Carnmenellis granite, the location of pressurised water was followed by means of microseismic detection of shearing on pre-existing joints. Initially surprisingly, it was found that the stimulated region grew in a downwards direction (Figure 17). Using a simple effective stress analysis of the type outlined above for the Jaeger simple plane of weakness theory, Pine and Batchelor (1984) found that this result could be explained in terms of the interactions of the anisotropic *in situ* stress field with critically aligned joints. The two-dimensional distinct element Fluid Rock Interaction Program (FRIP) developed as part of the CSM project (Pine and Cundall, 1985) was used to simulate this and other fluid injections. The downward growth was found to be closely related to the ratio of the maximum to minimum principal effective stress. The existence of the downward growth implies an eventual curvature in the maximum horizontal stress vs depth envelope if shear failures are not to be incipient under hydrostatic pore pressure conditions. This curvature implies that upward shear growth would occur during injections at greater depths (Pine and Batchelor, 1984).

One of the world's largest and most commercially attractive hot dry rock geothermal energy resources is currently being developed in the Cooper Basin, South Australia, by the Brisbane-based company, Geodynamics Limited. The conditions in the Cooper Basin which are well established from the large amount of petroleum exploration drilling carried out there over the last 30 years, are made favourable for HDR geothermal energy exploitation because of the 3 km cover of sedimentary rocks overlying and insulating the hot granites. At the time of writing, Geodynamics had drilled its first well to a depth of 4421m. It was planned that two hydraulic stimulations over vertical depths of 285 and 250m, respectively, would be carried out almost immediately. Because of the *in situ* stress field at the target depth, Geodynamics anticipated that the stimulated zone forming the underground heat exchanger would grow in a horizontal direction.

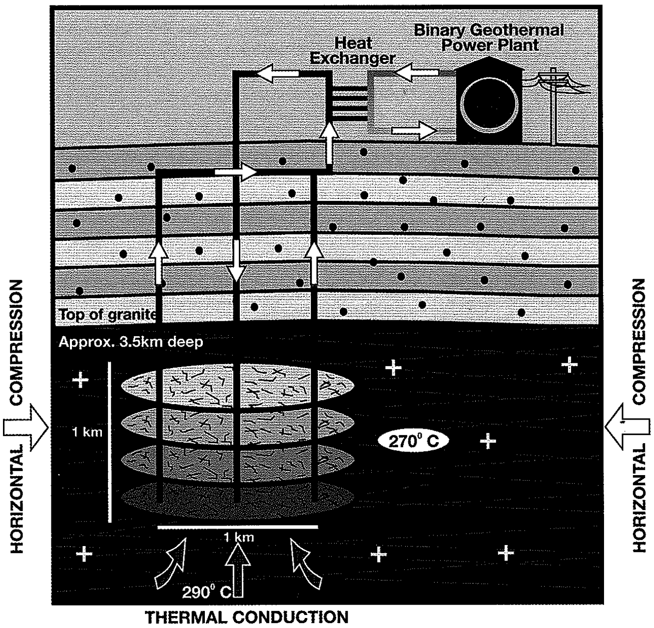


Figure 16: Conceptual Small Scale Hot Dry Rock Geothermal Energy Demonstration Plant (diagram by Geodynamics Ltd).

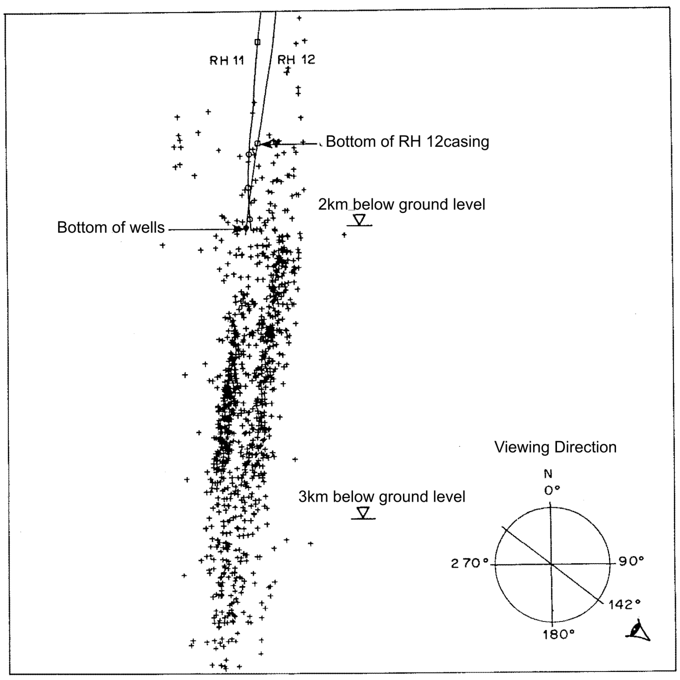


Figure 17: Vertical Section Showing Microseismic Locations Associated with Stimulation Injections, CSM Hot Dry Rock Geothermal Energy Project, Cornwall, UK (Pine and Batchelor, 1984).

10 CONCLUSIONS

Considerable advances have been made in the last 40 years in understanding the mechanics of discontinuous rock masses. Advances have also been made in the techniques available for site characterisation, geotechnical model formulation, design analyses and, although they haven't been discussed in any detail in this paper, in performance monitoring and back analysis as part of the rock engineering process. The advances made in numerical modelling capability have been particularly noteworthy. These advances have been used to outstanding effect in a range of applications in the civil engineering, mining and energy resource industries.

Despite the advances made in the state-of-the-art, the complex, variable and anisotropic nature of discontinuous rock masses, the difficulties of sampling and characterising them, and the difficulties of determining boundary conditions, particularly the *in situ* stresses, still pose a number of practical difficulties in engineering projects. Although they have not been discussed in any detail here, risk assessment and management techniques, and the use of numerical analyses to investigate controlling mechanisms and to carry out sensitivity studies, provide important tools in dealing with the uncertainties associated with engineering in discontinuous rock.

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